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MCDM approach for performance evaluation of the research institutions

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ABSTRACT

Evaluation of research institutions activities is one of the significant fields of scientometrics. We should note that in the previous approaches, the assessment of scientific institutions have been carried out mainly according to their scientific-theoretical activity. It is obvious that the activity of research institutions is not limited only to scientific-theoretical activity, their activity is multifarious. For this purpose, the paper first defines the system of criteria characterizing the activity of research institutions and then evaluates their activity based on these criteria. It is known that the importance degree of the criteria plays key role in multi-criteria evaluation. Thus, a new approach based on the "worst case" and "best case" methods was proposed for the calculation of criteria weights. VIKOR and TOPSIS methods were used for multi-criteria assessment of alternatives. The results of the experiments conducted in the study showed that the proposed approach demonstrates a stable result compared to others.

1. Introduction

Today, the assessment of scientists, journals, research institutions, and organizations has become more actual by the widespread usage of scientific indicators. Thus, in the assessment of any scientist, special attention is paid to his scientific works, the Impact Factor, CiteScore indicators of the journals in which his articles are published, and the h and g-index of the scientist. Currently, the evaluation issue of research institutions has become relevant. However, we should note that only scientific-theoretical activities of research institutions are considered during this evaluation (Stukalova, 2016).

The assessment of research institutions is a complicated issue that includes a decision-making process based on several criteria. Thus, accurate and reliable information about criteria and alternatives should be provided during the assessment process. In this assessment process, some traditional methods used in the assessment of research institutions are not enough. In the application of these methods, the decision-maker's conclusions based on the subjective opinion of the person sometimes call into question the accuracy of the results (Robertson & Smith, 2001, Sanaliyeva et al., 2021).

Multi-criteria decision making (MCDM) are used to select, evaluate, and rank contradicting alternatives for assessing the research institutions. MCDM problems involve the recruitment of employees (experience, age, familiarity with computers, salary agreement); job search (impressive job, close to home, future of the company, faster promotion); student rating (mathematics, physics, literature, English), etc. In other words, selecting the best option among

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several alternatives is considered a MCDM problem (Dursun & Karsak, 2010).

The paper aims to determine the system of criteria for assessing scientific institutions, and to calculate the weights of the criteria using the "worst case" and "best case" methods. These weights were calculated based on ranking and preference degree. The issue of multi-criteria assessment of alternatives via VIKOR and TOPSIS methods was analyzed by using calculated weights. In the end, we demonstrate the superiority of the proposed new method over other known methods.

2. Related work

Scientific institutions and organizations play a significant role in the development of science at the national and global level. Since universities, scientific institutions, and other institutions play a significant role in training highly qualified personnel. We can note that education is one of the main ways of increasing the country's competitiveness. In recent years, along with state universities, private educational institutions have also been promoting the development of the education system. Thus, the increase in the number of universities and scientific institutions has made the issue of their quality assessment urgent. For instance, the rapid increase in the number of universities in Taiwan in recent years has become a significant issue for the government, universities, and researchers in terms of improving the education system. Here the official mechanism used to assess and improve performance plays a key role in guiding the development of universities and government financial support. In particular, we can mention the significance of the quality of the teaching process and the selection of the teaching personnel.

Eliezer Geisler (1994) researched "Key Output Indicators in research and enterprise performance assessment". He provided detailed information on various indicators for evaluating the activities of his organizations - scientific results, innovation, intellectual property, financial and human resources, etc. and provided recommendations for their effective application. He argued the benefit of this approach both for the organizations themselves and for other stakeholders who monitor their results.

Hung et al. (2012) analyzed the assessment of the quality indicators of universities using the MCDM method. Thus, mentioning the significance of evaluating the quality and efficiency of universities, the ranking of 12 universities was analyzed by applying hybrid MCDM methods (AHP and TOPSIS). We determined that the proposed approach is a reliable and efficient method for evaluating university ratings. In addition, note that the proposed approach can be used in the process of making decisions in universities.

Nagesh R. and Vijayalakshmi S. (2014) analyzed and proposed a new analytical approach to evaluate the performance of scientific institutions and organizations to better understand their contribution to society and the economy and to compare and evaluate their performance. Also, they noted that the criteria and parameters used for the assessment of the activity in the research study play a significant role in the evaluation process. It should be noted that the assessment of the activity of scientific research institutions is more problematic. Because the scientific results obtained here are not material and measurable. This approach was proposed for objective assessment of personal profiles for each department (laboratory) of scientific institutions based on four weighting coefficients (public, private, social and strategic). The proposed analytical approach assists in the rational distribution of the data provided by the quantitative assessment of scientific institutions across departments.

Varmazyar and others (2016) used the Balanced Scorecard (BSC) strategic assessment tool, which considers both financial and non-financial indicators, to assess the business performance of organizations or companies. The paper proposed a new integrated approach based on BSC and four MCDM methods to assess the performance of research and technology research centers.

Fan Zong and Lifang Wang (2017) noted that the assessment of the university's research activity is significant for the results of published articles in the field of science and technology. The paper proposes a new D-AHP (Dynamic Analytic Hierarchy Process) approach using MCDM to evaluate university research performance. In the proposed approach, the main criteria characterizing the scientific-research activity of universities were defined and their relative importance was calculated. This is noted that D-AHP is a more accurate, objective, and flexible approach for evaluating the scientific and research activities of universities and allows for a detailed assessment of the scientific and research activities of universities.

In recent years, with the emergence of scientific indicators, we face multi-criteria assessment problems of research institutions. In this context, MCDM is used as a powerful means to assess and rank the alternatives based on frequently conflicting criteria (Chang, 2013; Chang, 2014).

3. Criteria for Evaluation

As mentioned above, this is incorrect to evaluate scientific institutions based on only one criterion in previous studies. For this reason, the following set of new criteria for the evaluation of scientific institutions and organizations has been included:

Scientific-theoretical activity – C_1. Scientific-theoretical activity refers to the scientific-research works of each institution.

Scientific-innovation and practical activity – C_2. Participation in grants and other innovative projects is intended.

Scientific-pedagogical activity – C_3. Scientific-pedagogical activity means the participation of employees in the teaching process at bachelor's, master's and other levels of education.

International scientific cooperation activity – C_4. The conduct of joint research with scientists working abroad is understood.

Scientific expertise activity – C_5. The participation in the expertise of research works and projects of various purposes is intended.

Promotion and popularization of scientific knowledge – C_6. The participation in the promotion and popularization of scientific knowledge in mass media and social media is intended.

Scientific-organizational activity – C_7. Meetings, local and international conferences, scientific seminars, etc. held by research institutions and organizations are considered.

4. The Modified Fuzzy VIKOR Method

We should note that the VIKOR method is used for multi-criteria optimization of complex systems. Via this method, alternatives are ranked and allow to make a final decision for issues with conflicting criteria (Yücenur and Demirel, 2012; Wan et. al, 2013, Hu et.al, 2014). Also, it increases the effectiveness of decisions by replacing the initial weights with the weights obtained during the compromise solution (Rostamzadeh et al., 2015). The extension of VIKOR (Opricovic, 2007) was proposed to determine the fuzzy compromise solution. In the paper, VIKOR and TOPSIS methods were used for multi-criteria assessment of research institutions. Therefore, the step-by-step explanation of VIKOR and TOPSIS methods is given below.

The VIKOR method consists of the following steps:

Step 1. Determination of the criteria for the evaluation of research institutions. At this stage, a brief description of the criteria used in the assessment of research institutions is given. Let's indicate the set of criteria for decision-making during the assessment of research institutions as $\{C_1, C_{2,\dots} C_m\}$.

Step 2. Establish a group of decision-makers. Let A_i $(i = 1, \dots, n)$ be a finite set of n alternatives which are to be evaluate by a group of K decision-makers DM_k $(k = 1, \dots, K)$ with respect to a set of m evulation criteria C_j $(j = 1, \dots, m)$.

Step 3. Identification of linguistic variables. At this stage, the weight of criteria and the relevant linguistic variables for assessing the alternatives for each criterion are determined. The linguistic variables given in Tables 1 and 2 are used to assess the importance of the criteria and alternatives to these criteria. The linguistic variables are represented as triangular fuzzy numbers.

Step 4. Construction of the evaluation matrix of research institutions. A typical fuzzy multi-criteria decision-making problem is expressed in matrix form as follows:

$$X_k = \|x_{ijk}\|$$

where x_{ijk} is the assessment of the k-th decision-maker, the assessment of the i-th alternative (A_i) with regard to the jth criterion (C_j) by the (DM_k). $x_{ijk} = (x_{ijk}^l, x_{ijk}^m, x_{ijk}^u)$ are linguistic variables.

Step 5. Calculation of aggregate fuzzy ratings of alternatives.

The aggregated fuzzy rating value of $x_{ijk} = (x_{ijk}^l, x_{ijk}^m, x_{ijk}^u)$ alternatives can be calculated as follows:

$$x_{ij}^l = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^l,\ x_{ij}^m = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^m,\ x_{ij}^u = \frac{1}{K}\sum_{k=1}^{K} x_{ijk}^u \quad (1)$$

Step 6. Calculation of fuzzy best and fuzzy worst values of criteria. The fuzzy best and fuzzy worst value are respectively defined as follows:

$$x_j^+ = \begin{cases} \max_{i=1,\dots,n}\{x_{ij}\}, & \text{for benefit criterion} \\ \min_{i=1,\dots,n}\{x_{ij}\}, & \text{for cost criterion} \end{cases} \quad (2)$$

$$x_j^- = \begin{cases} \min_{i=1,\dots,n}\{x_{ij}\}, & \text{for benefit criterion} \\ \max_{i=1,\dots,n}\{x_{ij}\}, & \text{for cost criterion} \end{cases} \quad (3)$$

where x_j^+ and x_j^- denote the fuzzy positive-ideal and the fuzzy negative-ideal solutions for jth criterion, respectively.

Table 1. The relative importance degree of criteria

Intensity of importance, R_j/R_q	Relative importance degree of criteria C_j and C_q	Description
1	Equally important	C_j criterion is equally important as criterion C_q
2	Weakly important	between 1 and 3
3	Average important	C_j criterion is slightly more important than criterion C_q
4	More important than average	Between 3 and 5
5	Strong important	C_j criterion is more important than criterion C_q
6	More important than strong	Between 5 and 7
7	Very strongly important	C_j criterion is strongly more important than criterion C_q
8	More important than very strong	Between 7 and 9
9	Extremely important	C_j criterion is much more important than the criterion C_q

Table 2. Linguistic variables for alternatives assessment

Linguistic variables	TFNs
Excellent	(8, 9, 10)
Good	(6, 7, 8)
Fair	(4, 5, 6)
Poor	(2, 3, 4)
Worst	(1, 1, 2)

Step 7. Calculation of benefit and modified cost indicators. In the VIKOR method, the following indicators are used to form the ranking measure:

$$S_i = \sum_{j=1}^{m} \left|\frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-}\right| \quad (4)$$

$$R_i^{MV} = \sum_{j=1}^{m} \left|\frac{x_{ij} - x_j^-}{x_j^+ - x_j^-}\right|, i = 1, \dots, n. \quad (5)$$

where S_i və R_i^{MV} indicate the benefit and modified cost indicators, respectively. S_i is the distance from A_i to the positive ideal solution, and R_i^{MV} is the distance from A_i to the negative ideal solution.

Step 8. Calculation of Q_i values. Q_i^V VIKOR index is used for the ranking of the alternatives:

$$Q_i^V = \lambda \frac{S_i - S^-}{S^+ - S^-} + (1-\lambda)\frac{R_i^{MV} - R^{MV-}}{R^{MV+} - R^{MV-}}, i = 1, \dots, n. \quad (6)$$

where,

$$S^+ = \max_{i=1,\dots,n}\{S_i\},\ S^- = \min_{i=1,\dots,n}\{S_i\}, \quad (7)$$

$$R^{\mathrm{MV}+} = \max_{i=1,\dots,n}\{R_i^{\mathrm{MV}}\}$$

and $$R^{MV-} = \min_{i=1,\dots,n}\{R_i^{\mathrm{MV}}\} \quad (8)$$

The solution obtained by S^+ refers to the maximum group utility ("majority" rule), while the solution obtained by R^+ refers to the minimum value of the individual "opponent", and $\lambda \epsilon [0,1]$ is the weight of the decision-making strategy “the majority of criteria” (or “the maximum group utility”).

Step 9. Defuzzification. The most commonly used centroid method (**Lee, 1990; Sugeno, 1985**) is used for defuzzification. Using the triangular fuzzy Eq., the defuzzified value of $A = (a^l, a^m, a^u)$ is calculated as follows:

$$A = \frac{a^l + a^m + a^u}{3} \quad (9)$$

Step 10. Ranking of alternatives. The alternatives are ranked by arranging the values of Q, S and R in ascending order. The smaller the value of Q the better the alternative is considered.

Step 11. Proposing a compromise solution. If the following two conditions are satisfied, then the minimum-value scheme Q is considered the optimal compromise solution for ranking (Opricovic, 2011).

Perceived advantage: if $\frac{Q(A^{(2)}) - Q(A^{(1)})}{Q(A^{(n)}) - Q(A^{(1)})} \geq \frac{1}{n-1}$, then alternative $A^{(1)}$ has the perceived advantage. where $A^{(1)}$ is the best alternative, $A^{(2)}$- is the alternative in the second position, $A^{(n)}$ is the alternative in the last position, and n is the number of alternatives.

Perceived stability: Alternative $A^{(1)}$ should have the best ranking in terms of dimensions S or

R. A compromise solution is considered stable in the following cases in the decision-making process: "by majority vote" (it should be $\lambda > 0.5$) or "by consensus" should be ($\lambda = 0.5$) and by "veto" should be ($\lambda < 0.5$).

5. TOPSIS method

The TOPSIS method consists of the following steps:

As mentioned above, in the TOPSIS method, the Euclidean distance of each alternative is calculated from the positive and negative ideal solution.

Step 1. Finding the best and worst ideal solution. For each ideal alternative, the best (V^+) and the worst (V^-) performance are determined.

$$V^+ = \{v_1^+, v_2^+, \dots, v_m^+\} = \{\max_{i=1,\dots,n} v_{ij} \text{ for } \forall j \in 1, \dots, m\} \quad (10)$$

$$V^- = \{v_1^-, v_2^-, \dots, v_m^-\} = \{\min_{i=1,\dots,n} v_{ij} \text{ for } \forall j \in 1, \dots, m\} \quad (11)$$

Step 2. Determination of separation values. Separation values are the distance of each alternative from both positive and negative ideal solutions, obtained by applying Euclidean distance. In other words, the distance to the best alternative (D_i^+) and the distance to the worst alternative (D_i^-) are calculated for all alternatives by using the Eqs. (10)-(11):

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}, i = 1, \dots, n \quad (12)$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, i = 1, \dots, n \quad (13)$$

Step 3. Calculation of the proximity of each alternative ($D_i^+$and D_i^-) **to the negative ideal solution.** This proximity is calculated as follows:

$$V_i = \frac{D_i^-}{D_i^+ + D_i^-}, i = 1, \dots, n \quad (14)$$

where $0 \le V_i \le 1$, $V_i = 1$ corresponds to the best solution, and $V_i = 0$ to the worth solution.

Best value V_i is considered as the best option and the solution for the MCDM problem through TOPSIS.

6. Calculation of the criteria weights: The "worst case" and the "best-worst case" method

6.1. The idea of the "worst case" method (Rotshtein, 2009) is derived from the structural analysis of the system. Here, the reliability of the system is distributed among its elements according to their ranking. The higher the ranking, the more reliable it will be. Previous methods (Hwang & Yoon, 1981; Saaty, 2006; Patil & Kant, 2014; Lin, 2010) used the pairwise comparison method to determine the weight of the criteria. In contrast, in the proposed method, the criteria are compared only with the most important criterion.

Suppose that, w_j^k is the weight given to the jth criterion (C_j) by the kth decision-maker (DM_k) who expresses the importance of the criterion. Let's assume that, C_j criterion has the largest weight w_j^k and the highest ranking R_j^k. This can be expressed as follows:

$$\frac{w_1^k}{R_1^k} = \frac{w_2^k}{R_2^k} = \dots = \frac{w_q^k}{R_q^k} = \dots = \frac{w_m^k}{R_m^k}, k = 1,2, \dots, K. \quad (15)$$

Suppose that the weight and rank of the least important criterion assessed by the decision maker DM_k are denoted by w_q^k and R_q^k , respectively. From Eq. (1), we obtain the following expression for the weight of the criteria considering the least important criterion assessed by the decision-maker DM_k:

$$w_1^k = R_1^k \frac{w_q^k}{R_q^k}, w_2^k = R_2^k \frac{w_q^k}{R_q^k}, \dots,$$

$$w_m^k = R_m^k \frac{w_q^k}{R_q^k}, k = 1, \dots, K. \quad (16)$$

Assume that the following condition is satisfied:

$$w_1^k + w_2^k + \dots + w_q^k + \dots + w_m^k = 1, \; k = 1,2, \dots, K. \quad (17)$$

Substituting expressions (15) and (16) into (17), we obtain the following expression for the weight of the least important criterion:

$$w_q^k = \frac{1}{\frac{R_1^k}{R_q^k} + \frac{R_2^k}{R_q^k} + \dots + \frac{R_q^k}{R_q^k}} = \frac{1}{\sum_{j=1}^{m} \frac{R_j^k}{R_q^k}}, \; k = 1, \dots, K. \quad (18)$$

Eqs. (16) and (18) enable to calculation of the weight of the criteria using the ratio of the ranking of all C_j criteria to the ranking of the least important criterion. We should note that the comparison with the worst (less important) case guarantees that condition $\frac{R_j^k}{R_q^k} \ge 1$ is satisfied for all $j = 1,2, \dots, m$ and $k = 1,2, \dots, K$. The ranking ratio ($\frac{R_j^k}{R_q^k}$) of the criteria in Eq. (18) is evaluated using Saaty's 1-9 scale given in Table 1 (Saaty, 2006; Saaty, 2008).

Using Eq. (1), the following aggregate weights of criteria are obtained through Eqs. (16) and (18):

$$w_j = \frac{1}{K} \sum_{k=1}^{K} w_j^k, j = 1,2, \dots, m. \quad (19)$$

6.2. The "best-worst case" method (Rezaei, 2015). A scale of 1/9 to 9 (Saaty, 2006) is used for pairwise comparison of these criteria. The comparison matrix will be as follows:

$$B = \begin{pmatrix} b_{11} & b_{12} & \dots b_{1m} \\ b_{21} & b_{22} & \dots b_{2m} \\ \vdots & \vdots & \vdots\ \vdots \\ b_{m1} & b_{m2} & \dots b_{nm} \end{pmatrix}$$

Here b_{ij} is the relative advantage of criteria i over the criteria. $b_{ij} = 1$ indicates that criteria i and j have the same importance. The case $b_{ij} = 9$ indicates that ith criterion is more important than jth criterion. The preference of j over i is expressed by b_{ji}. For matrix B to be reciprocal, the condition $b_{ij} = 1/b_{ji}$ and $B_{ii} = 1$ should be satisfied for all i and j. Considering the inverse feature of the B matrix, $m(m-1)/2$ number of pairwise comparisons are required to obtain the completed B matrix. A pairwise comparison must correspond to the matrix B by satisfying the condition for $b_{ik} * b_{kj} = b_{ij}, \forall i, j$. As seen, pairwise comparison is the basis of the "best-worst case" method. When b_{ij} pairwise comparison is performed, the expert should state the preference of the ith criterion over the jth criterion.

7. Modified "worst case" and "best case" methods

7.1. Ranking of the "worst case" and "best case" criteria according to the preference degree

The "worst case" and "best case" criteria are determined by each expert. b_{ij} element is evaluated between 1 and 9. Then these criteria with other criteria are compared in pairs. The preference of other criteria over the "worst case" criterion is determined by experts. By the same way, the priority of the "best case" criteria over other criteria is determined. Criteria are ranked according to the preference degree. Using rankings for "worst case" and "best case" (Aliguliyev, 2009; Alguliyev, Aliguliyev & Mahmudova, 2015)

$$RR = \sum_{j=1}^{m} \frac{(m-j+1)r_j}{m} \quad (20)$$

a final ranking is calculated for each criteria.

The comparison vector in regard to the "best" criterion is expressed as follows:

$$B_{\text{Best}}^{RR} = (b_{1,\text{RR}}^{\text{best}}, b_{2,\text{RR}}^{\text{best}}, \dots, b_{m,\text{RR}}^{\text{best}}) \quad (21)$$

where $b_{j,\text{RR}}^{\text{best}}$ is the preference degree of the best ($best$) criteria over the j-th criteria. It is obvious that, $\exists j^+ \in \{1,2,\dots,m\}$, $b_{j^+}^{\text{best}} = 1$.

$$w_{j,\text{RR}}^{\text{best}} = w_{RR}^{\text{best}} \frac{R_j}{R_{\text{best}}} = w_{RR}^{\text{best}} * k_{j,RR}^{\text{best}},\ j = 1,2,\dots,m \quad (22)$$

where $k_{j,RR}^{\text{best}} = \frac{\text{R}_\text{j}}{\text{R}_{\text{best}}}$ – is the relative ranking of j-th criteria according to the best criterion, and w^{best} is the weight of the best criterion:

$$w_{RR}^{\text{best}} = \frac{1}{\sum_{j=1}^{m} \frac{1}{k_{j,\text{PD}}^{\text{best}}}} \quad (23)$$

According to the "worst" criterion, the comparison vector is expressed as follows:

$$B_{\text{Worst}}^{RR} = \left(b_{1,RR}^{\text{worst}}, b_{2,RR}^{\text{worst}}, \dots, b_{m,RR}^{\text{worst}}\right)^T \quad (24)$$

where b_j^{worst} indicates the preference of the j-th criterion in regard to the worst criterion. It is obvious that, $\exists\ j^- \in \{1,2,\dots,m\}$, $b_{j-,RR}^{worst} = 1$.

Similarly, to the "best case" method, the weights of criteria for the "worst case" method are calculated as follows:

$$w_{j,RR}^{\text{worst}} = w_{RR}^{\text{worst}} \frac{R_j}{R_{\text{worst}}} = w_{j,RR}^{\text{worst}} * k_{j,\text{RR}}^{\text{worst}},\ j = 1,\dots,m \quad (25)$$

where $k_{j,\text{RR}}^{\text{worst}} = \frac{R_j}{R_{\text{worst}}}$ - is the relative ranking of j-th criterion in regard to the worst criterion, w_{worst} – is the weight of the worst criterion:

$$w_{RR}^{\text{worst}} = \frac{1}{\sum_{j=1}^{m} k_{j,\text{RR}}^{\text{worst}}} \quad (26)$$

The final weights of the criteria are calculated as follows:

$$w_{j,\text{RR}} = \frac{w_{\text{j,RR}}^{\text{worst}} + w_{\text{j,RR}}^{\text{best}}}{2} \quad (27)$$

$$\sum_{j=1}^{m} w_{j,\text{RR}} = 1, \text{ for all } w_j \geq 0.$$

7.2. Determination of the "worst case" and "best case" criteria based on the degree of preference:

In the proposed approach, the "worst case" and the "best case" criteria are determined by each expert. b_{ij} element is evaluated between 1 and 9. Then these criteria with other criteria are compared in pairs. The preference of other criteria over the "worst case" criterion is determined by experts. By the same way, the priority of the "best case" criteria over other criteria is determined.

The comparison vector according to the "best" criteria is expressed as follows:

$$B_{\text{best}}^{PD} = (b_{1,\text{PD}}^{\text{best}}, b_{2,\text{PD}}^{\text{best}}, \dots, b_{m,\text{PD}}^{\text{best}}) \quad (28)$$

where $b_{j,\text{PD}}^{\text{best}}$ indicates the preference of the best criteria ($best$) over the j-th criteria. It is obvious that, $\exists j^+ \in \{1,2,\dots,m\}$, $b_{j^+}^{\text{best}} = 1$.

$$w_{j,\text{PD}}^{\text{best}} = w_{\text{PD}}^{\text{best}} \frac{R_j}{R_{\text{best}}} = w_{\text{PD}}^{\text{best}} * k_{j,PD}^{\text{best}},\ j = 1,\dots,m \quad (29)$$

Here $k_{j,PD}^{\text{best}} = \frac{\text{R}_\text{j}}{\text{R}_{\text{best}}}$ – is the relative ranking of j-th criteria in regard to the best criteria, and w^{best} is the weight of the best criteria:

$$w_{\text{PD}}^{\text{best}} = \frac{1}{\sum_{j=1}^{m} \frac{1}{k_{j,\text{PD}}^{\text{best}}}} \quad (30)$$

According to the "worst" criteria, the comparison vector is expressed as follows:

$$B_{\text{Worst}}^{\text{PD}} = \left(b_{1,\text{PD}}^{\text{worst}}, b_{2,\text{PD}}^{\text{worst}}, \dots, b_{m,\text{PD}}^{\text{worst}}\right)^T \quad (31)$$

where b_j^{worst} indicates the prominence of the j-th criteria according to the worth ($Worst$) criteria. It is obvious that, ∃ $j^- \in \{1,2,\dots,m\}$, $b_{j-,\text{PD}}^{worst} = 1$.

Similarly, to the "best case" method, the weight of criteria for the "worst case" method is calculated as follows:

$$w_{j,\text{PD}}^{\text{worst}} = w_{\text{PD}}^{\text{worst}} \frac{R_j}{R_{\text{worst}}} = w_{j,\text{PD}}^{\text{worst}} * k_{j,\text{PD}}^{\text{worst}}, j = 1,\dots,m \quad (32)$$

where $k_{j,\text{PD}}^{\text{worst}} = \frac{R_j}{R_{\text{worst}}}$ - is the relative rank of the j-th criterion according to the worst criterion, w_{worst} – is the weight of the worst criterion:

$$w_{\text{PD}}^{\text{worst}} = \frac{1}{\sum_{j=1}^{m} k_{j,\text{PD}}^{\text{worst}}} \quad (33)$$

The final weights of criteria are calculated as the average of the weights calculated by both methods:

$$w_{\text{j,PD}} = \frac{w_{\text{j,PD}}^{\text{worst}} + w_{\text{j,PD}}^{\text{best}}}{2} \quad (34)$$

$$\sum_{j=1}^{m} w_{j,\text{PD}} = 1, \text{ for all } w_j \geq 0.$$

8. Experimental assessment

Experiments were conducted on empirical data to evaluate the proposed method. Let's assume that 10 scientific research institutions ($\text{A}_i, i = 1,\dots,10$) were selected for evaluation. An expert board consisting of five independent decision-makers ($\text{DM}_k, k = 1,\dots,5$) was established to evaluate the enterprises according to the criteria given above ($\text{C}_j, j = 1,\dots,7$). First, the "best" and "worst" criteria were determined by each expert, and then the importance degree of the criteria were determined for each case (Tables 3-4). Based on Tables 3-4, the final ranking of the criteria was conducted (Tables 5-6). In Tables 3 and 4, the ranking values of the criteria are shown in parentheses PD_{bj} and PD_{jw}, $j = 1,\dots,7$.

Where PD_{bj} is the preference degree of the best (b) criterion over jth criterion, $j = 1,\dots,7$, $b \in \{4,5,1,2,1\}$.

Table 3. Pairwise comparison vector for the best criterion (preference degree)

Expert	Best criterion	Criteria													
		C_1		C_2		C_3		C_4		C_5		C_6		C_7	
		PD_{b1}		PD_{b2}		PD_{b3}		PD_{b4}		PD_{b5}		PD_{b6}		PD_{b7}	
DM1	C_4	3	(5)	7	(1)	2	(6)	1	(7)	6	(2)	5	(3)	4	(4)
DM2	C_5	8	(1)	7	(2)	3	(5)	2	(6)	1	(7)	6	(3)	4	(4)
DM3	C_1	1	(7)	8	(4)	3	(6)	5	(3)	2	(1)	7	(5)	4	(2)
DM4	C_2	7	(1)	1	(7)	5	(5)	6	(4)	3	(6)	4	(3)	7	(2)
DM5	C_1	1	(7)	7	(2)	8	(1)	3	(5)	5	(3)	2	(6)	4	(4)

Table 4. Pairwise comparison vector for the worst criterion (preference degree)

Criteria	Worst criterion									
	DM1		DM2		DM3		DM4		DM5	
	C_2		C_1		C_2		C_1		C_3	
	PD_{j2}		PD_{j5}		PD_{j1}		PD_{j2}		PD_{j3}	
C_1	4	(4)	1	(7)	8	(1)	1	(7)	8	(1)
C_2	1	(7)	4	(4)	1	(7)	7	(1)	3	(5)
C_3	3	(5)	2	(6)	4	(5)	5	(3)	1	(7)
C_4	7	(1)	5	(3)	5	(4)	4	(4)	7	(2)
C_5	5	(3)	8	(1)	3	(6)	3	(5)	6	(3)
C_6	6	(2)	3	(5)	6	(3)	6	(2)	2	(6)
C_7	2	(6)	6	(2)	7	(2)	2	(6)	5	(4)

PD_{jw} is the preference degree of the jth criterion over the worst (w) criterion, $j = 1,\dots,7$, $w \in \{2,5,1,2,3\}$.

In Table 3, ranks are defined by rows, and in Table 4, by columns. The ranks of the criteria obtained from Table 3 are given in Table 5, the ranks of the criteria obtained from Table 4 are given in Table 6. r_p in the last column of Table 5 shows the number of occurrences of the criterion in the p-th position (rank). For instance, $r_7 = 2$ means that the criterion is found twice in position 7.

Creation of the initial decision matrix: As mentioned above, experts are required to evaluate the alternatives according to the criteria $C_1 \div C_7$, using the linguistic terms given in Tables 1-2. The experts' decision matrices are given in Tables 7-11.

Table 5. The ranking criteria obtained by the "Best case" method (Preference Degree)

Criteria	DM1	DM2	DM3	DM4	DM5	Best
C_1	5	1	7	1	7	$r_1 = 2,\ r_5 = 1, r_7 = 2$
C_2	7	2	1	7	2	$r_1 = 1,\ r_2 = 2, r_7 = 2$
C_3	6	5	5	3	1	$r_1 = 1,\ r_3 = 1, r_5 = 2, r_6 = 1$
C_4	1	6	3	2	5	$r_1 = 1,\ r_2 = 1, r_3 = 1, r_5 = 1, r_6 = 1$
C_5	2	7	6	5	3	$r_2 = 1,\ r_3 = 1, r_5 = 1, r_6 = 1, r_7 = 1$
C_6	3	3	2	4	6	$r_2 = 1, r_3 = 1, r_4 = 1, r_6 = 1$
C_7	4	4	4	6	4	$r_4 = 4,\ r_6 = 1$

Table 6. Ranking criteria obtained by the "Worst case" method (Preference Degree)

Criteria	DM1	DM2	DM3	DM4	DM5	Worst
C_1	4	7	1	7	1	$r_1 = 2,\ r_4 = 1, r_7 = 2$
C_2	7	4	7	1	5	$r_1 = 1,\ r_4 = 1, r_5 = 1, r_7 = 2$
C_3	5	6	5	3	7	$r_3 = 1, r_5 = 2, r_6 = 1, r_7 = 1$
C_4	1	3	4	4	2	$r_1 = 1,\ r_2 = 1, r_3 = 1, r_4 = 2$
C_5	3	1	6	5	3	$r_1 = 1,\ r_3 = 2, r_5 = 1, r_6 = 1$
C_6	2	5	3	2	6	$r_2 = 2,\ r_3 = 1, r_5 = 1, r_6 = 1$
C_7	6	2	2	6	4	$r_2 = 2,\ r_4 = 1\ r_6 = 2$

Table 7. Fuzzy decision matrix of DM1

Alternatives	Criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	(4,5,6)	(1,1,2)	(6,7,8)	(2,3,4)	(4,5,6)	(6,7,8)	(8,9,10)
A_2	(8,9,10)	(6,7,8)	(2,3,4)	(4,5,6)	(8,9,10)	(1,1,2)	(6,7,8)
A_3	(4,5,6)	(8,9,10)	(2,3,4)	(6,7,8)	(2,3,4)	(1,1,2)	(2,3,4)
A_4	(2,3,4)	(6,7,8)	(1,1,2)	(4,5,6)	(4,5,6)	(2,3,4)	(1,1,2)
A_5	(2,3,4)	(1,1,2)	(2,3,4)	(1,1,2)	(2,3,4)	(2,3,4)	(1,1,2)
A_6	(1,1,2)	(2,3,4)	(6,7,8)	(6,7,8)	(4,5,6)	(6,7,8)	(1,1,2)
A_7	(1,1,2)	(8,9,10)	(2,3,4)	(2,3,4)	(1,1,2)	(6,7,8)	(1,1,2)
A_8	(2,3,4)	(4,5,6)	(6,7,8)	(8,9,10)	(4,5,6)	(1,1,2)	(8,9,10)
A_9	(4,5,6)	(2,3,4)	(1,1,2)	(2,3,4)	(2,3,4)	(6,7,8)	(4,5,6)
A_{10}	(6,7,8)	(2,3,4)	(8,9,10)	(1,1,2)	(4,5,6)	(8,9,10)	(6,7,8)

Table 8. Fuzzy decision matrix of DM2

Alternatives	Criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	(8,9,10)	(6,7,8)	(2,3,4)	(4,5,6)	(8,9,10)	(1,1,2)	(6,7,8)
A_2	(6,7,8)	(8,9,10)	(4,5,6)	(1,1,2)	(4,5,6)	(2,3,4)	(1,1,2)
A_3	(8,9,10)	(6,7,8)	(1,1,2)	(2,3,4)	(1,1,2)	(6,7,8)	(2,3,4)
A_4	(2,3,4)	(6,7,8)	(1,1,2)	(4,5,6)	(4,5,6)	(2,3,4)	(1,1,2)
A_5	(8,9,10)	(4,5,6)	(2,3,4)	(6,7,8)	(4,5,6)	(6,7,8)	(2,3,4)
A_6	(4,5,6)	(2,3,4)	(6,7,8)	(6,7,8)	(4,5,6)	(6,7,8)	(1,1,2)
A_7	(8,9,10)	(1,1,2)	(4,5,6)	(2,3,4)	(1,1,2)	(4,5,6)	(2,3,4)
A_8	(4,5,6)	(8,9,10)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(2,3,4)
A_9	(6,7,8)	(6,7,8)	(1,1,2)	(1,1,2)	(2,3,4)	(8,9,10)	(6,7,8)
A_{10}	(4,5,6)	(8,9,10)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(2,3,4)

Table 9. Fuzzy decision matrix of DM3

Alternatives	Criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	(2,3,4)	(6,7,8)	(2,3,4)	(4,5,6)	(8,9,10)	(1,1,2)	(6,7,8)
A_2	(1,1,2)	(2,3,4)	(8,9,10)	(4,5,6)	(1,1,2)	(6,7,8)	(4,5,6)
A_3	(4,5,6)	(6,7,8)	(1,1,2)	(6,7,8)	(4,5,6)	(2,3,4)	(6,7,8)
A_4	(8,9,10)	(1,1,2)	(6,7,8)	(8,9,10)	(2,3,4)	(6,7,8)	(8,9,10)

A_5	(8,9,10)	(6,7,8)	(8,9,10)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)
A_6	(4,5,6)	(8,9,10)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(2,3,4)
A_7	(6,7,8)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(8,9,10)	(1,1,2)
A_8	(1,1,2)	(2,3,4)	(8,9,10)	(4,5,6)	(1,1,2)	(6,7,8)	(4,5,6)
A_9	(6,7,8)	(6,7,8)	(8,9,10)	(1,1,2)	(2,3,4)	(8,9,10)	(6,7,8)
A_{10}	(4,5,6)	(8,9,10)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(2,3,4)

Table 10. Fuzzy decision matrix of DM4

Alternatives	Criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	(6,7,8)	(8,9,10)	(2,3,4)	(4,5,6)	(2,3,4)	(6,7,8)	(1,1,2)
A_2	(2,3,4)	(1,1,2)	(8,9,10)	(6,7,8)	(1,1,2)	(8,9,10)	(4,5,6)
A_3	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(2,3,4)	(4,5,6)	(6,7,8)
A_4	(1,1,2)	(8,9,10)	(8,9,10)	(1,1,2)	(4,5,6)	(1,1,2)	(8,9,10)
A_5	(8,9,10)	(1,1,2)	(2,3,4)	(6,7,8)	(8,9,10)	(2,3,4)	(6,7,8)
A_6	(1,1,2)	(8,9,10)	(2,3,4)	(2,3,4)	(1,1,2)	(6,7,8)	(1,1,2)
A_7	(4,5,6)	(2,3,4)	(6,7,8)	(1,1,2)	(4,5,6)	(1,1,2)	(4,5,6)
A_8	(1,1,2)	(2,3,4)	(8,9,10)	(4,5,6)	(1,1,2)	(6,7,8)	(6,7,8)
A_9	(6,7,8)	(6,7,8)	(8,9,10)	(1,1,2)	(2,3,4)	(8,9,10)	(6,7,8)
A_{10}	(4,5,6)	(8,9,10)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(2,3,4)

Table 11. Fuzzy decision matrix of DM5

Alternatives	Criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	(8,9,10)	(1,1,2)	(2,3,4)	(6,7,8)	(8,9,10)	(2,3,4)	(6,7,8)
A_2	(4,5,6)	(8,9,10)	(1,1,2)	(6,7,8)	(4,5,6)	(1,1,2)	(2,3,4)
A_3	(2,3,4)	(6,7,8)	(2,3,4)	(4,5,6)	(2,3,4)	(6,7,8)	(1,1,2)
A_4	(6,7,8)	(8,9,10)	(4,5,6)	(2,3,4)	(1,1,2)	(6,7,8)	(8,9,10)
A_5	(2,3,4)	(6,7,8)	(1,1,2)	(4,5,6)	(4,5,6)	(2,3,4)	(1,1,2)
A_6	(4,5,6)	(2,3,4)	(4,5,6)	(8,9,10)	(4,5,6)	(6,7,8)	(2,3,4)
A_7	(6,7,8)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(8,9,10)	(1,1,2)
A_8	(4,5,6)	(8,9,10)	(2,3,4)	(6,7,8)	(4,5,6)	(1,1,2)	(2,3,4)
A_9	(2,3,4)	(6,7,8)	(8,9,10)	(1,1,2)	(2,3,4)	(8,9,10)	(2,3,4)
A_{10}	(4,5,6)	(1,1,2)	(8,9,10)	(6,7,8)	(1,1,2)	(8,9,10)	(4,5,6)

Aggregation of decision matrices: Based on the fuzzy decision matrices given in Tables 7-11, the aggregated fuzzy decision matrix is calculated (Table 12).

Note that the aggregation of fuzzy values was carried out with the help of the Eq. (19).

In addition, positive ideal X_j^+ and negative ideal X_j^- solutions are defined. Fuzzy positive ideal solution X_j^+ and fuzzy negative ideal solution X_j^- were calculated using Eqs. (10) and (11).

Calculation of criteria weights. The experts determined the least important criterion and its rank accordingly to calculate the weights of the criteria using the "worst case" method. Then, using Saaty's table, they ranked the other criteria relative to the least important criterion. The ranking criteria assigned by each expert is listed in Table 5 and Table 6.

Using the Eq. (20), the final ranking of the criteria for each case (Best and Worst) is calculated. The final rankings of the criteria according to Table 5 are given below:

Table 12. Aggregated fuzzy decision matrix

Alternatives	Criteria						
	C1	C2	C3	C4	C5	C6	C7
A_1	(5.6, 6.6, 7.6)	(4.4, 5.0, 6.0)	(2.8, 3.8, 4.8)	(4.0, 5.0, 6.0)	(6.0, 7.0, 8.0)	(3.2, 3.8, 4.8)	(5.4, 6.2, 7.2)
A_2	(4.2, 5.0, 6.0)	(5.0, 5.8, 6.8)	(4.6, 5.4, 6.4)	(4.2, 5.0, 6.0)	(3.6, 4.2, 5.2)	(3.6, 4.2, 5.2)	(3.4, 4.2, 5.2)
A_3	(4.0, 5.0, 6.0)	(6.4, 7.4, 8.4)	(2.0, 2.6, 3.6)	(3.8, 4.6, 5.6)	(2.2, 3.0, 4.0)	(3.8, 4.6, 5.6)	(3.4, 4.2, 5.2)
A_4	(3.8, 4.6, 5.6)	(5.8, 6.6, 7.6)	(4.0, 4.6, 5.6)	(3.8, 4.6, 5.6)	(3.0, 3.8, 4.8)	(3.4, 4.2, 5.2)	(5.2, 5.8, 6.8)
A_5	(5.6, 6.6, 7.6)	(3.6, 4.2, 5.2)	(3.0, 4.0, 5.0)	(3.8, 4.6, 5.6)	(4.8, 5.8, 6.8)	(3.2, 4.2, 5.2)	(2.2, 2.6, 3.6)
A_6	(2.8, 3.4, 4.4)	(4.4, 5.4, 6.4)	(4.0, 5.0, 6.0)	(5.6, 6.6, 7.6)	(3.4, 4.2, 5.2)	(5, 5.8, 6.8)	(1.4, 1.8, 2.8)
A_7	(5.0, 5.8, 6.8)	(3.0, 3.8, 4.8)	(4.8, 5.8, 6.8)	(2.6, 3.4, 4.4)	(1.6, 1.8, 2.8)	(5.4, 6.2, 7.2)	(1.8, 2.2, 3.2)

A_8	(2.4, 3.0, 4.0)	(4.8, 5.8, 6.8)	(5.2, 6.2, 7.2)	(5.6, 6.6, 7.6)	(2.8, 3.4, 4.4)	(3.0, 3.4, 4.4)	(4.4, 5.4, 6.4)
A_9	(4.8, 5.4, 6.4)	(5.2, 6.2, 7.2)	(5.2, 5.8, 6.8)	(1.2, 1.4, 2.4)	(2.0, 3.0, 4.0)	(7.6, 8.6, 9.6)	(4.8, 5.8, 6.8)
A_{10}	(4.4, 5.4, 6.4)	(5.4, 6.2, 7.2)	(4.4, 5.4, 6.4)	(5.0, 5.8, 6.8)	(3.4, 4.2, 5.2)	(3.8,4.2, 5.2)	(3.2, 4.2, 5.2)
X^+	(5.6, 6.6, 7.6)	(6.4, 7.4, 8.4)	(5.2, 6.2, 7.2)	(5.6, 6.6, 7.6)	(6.0, 7.0, 8.0)	(7.6, 8.6, 9.6)	(5.4, 6.2, 7.2)
X^-	(2.4, 3.0, 4.0)	(3.0, 3.8, 4.8)	(2.0, 2.6, 3.6)	(1.2, 1.4, 2.4)	(1.6, 1.8, 2.8)	(3.0, 3.4, 4.4)	(1.4, 1.8, 2.8)

$$RR^{\text{best}}(\mathrm{C}_1)=\sum_{\mathrm{j}=1}^{7}\frac{(7-\mathrm{j}+1)\mathrm{r_j}}{7}=\sum_{\mathrm{j}=1}^{7}\frac{(8-\mathrm{i})\mathrm{r_i}}{7}=\frac{(8-1)*2}{7}+\frac{(8-5)*1}{7}+\frac{(8-7)*2}{7}=\frac{19}{7}=2.714$$

$$RR^{\text{best}}(\mathrm{C}_2)=\frac{(8-1)*1}{7}+\frac{(8-2)*2}{7}+\frac{(8-7)*2}{7}=\frac{21}{7}=3.000$$

$$RR^{\text{best}}(\mathrm{C}_3)=\frac{(8-1)*1}{7}+\frac{(8-3)*1}{7}+\frac{(8-5)*2}{7}+\frac{(8-6)*1}{7}=\frac{20}{7}=2.857$$

$$RR^{\text{best}}(\mathrm{C}_4)=\frac{(8-1)*1}{7}+\frac{(8-2)*1}{7}+\frac{(8-3)*1}{7}+\frac{(8-5)*1}{7}+\frac{(8-6)*1}{7}=\frac{23}{7}=3.285$$

$$RR^{\text{best}}(\mathrm{C}_5)=\frac{(8-2)*1}{7}+\frac{(8-3)*1}{7}+\frac{(8-6)*1}{7}+\frac{(8-7)*1}{7}=\frac{17}{7}=2.429$$

$$RR^{\text{best}}(\mathrm{C}_6)=\frac{(8-2)*1}{7}+\frac{(8-3)*2}{7}+\frac{(8-4)*1}{7}+\frac{(8-6)*1}{7}=\frac{22}{7}=3.143$$

$$RR^{\text{best}}(\mathrm{C}_7)=\frac{(8-4)*4}{7}+\frac{(8-6)*1}{7}=\frac{18}{7}=2.571$$

The weights criteria are calculated using the final ranks above (Table 13):

$$\mathrm{w}_{1,\mathrm{RR}}^{\text{best}}=\frac{2.714}{2.714+3.000+2.857+3.286+2.429+3.143+2.571}=\frac{2.714}{20}=0.1357$$

$$\mathrm{w}_{2,\mathrm{RR}}^{\text{best}}=\frac{3.000}{2.714+3.000+2.857+3.286+2.429+3.143+2.571}=\frac{3.000}{20}=0.1500$$

$$\mathrm{w}_{3,\mathrm{RR}}^{\text{best}}=\frac{2.857}{2.714+3.000+2.857+3.286+2.429+3.143+2.571}=\frac{2.857}{20}=0.1429$$

$$\mathrm{w}_{4,\mathrm{RR}}^{\text{best}}=\frac{3.286}{2.714+3.000+2.857+3.286+2.429+3.143+2.571}=\frac{3.286}{20}=0.1643$$

$$\mathrm{w}_{5,\mathrm{RR}}^{\text{best}}=\frac{2.429}{2.714+3.000+2.857+3.286+2.429+3.143+2.571}=\frac{2.429}{20}=0.1214$$

$$\mathrm{w}_{6,\mathrm{RR}}^{\text{best}}=\frac{3.143}{2.714+3.000+2.857+3.286+2.429+3.143+2.571}=\frac{3.143}{20}=0.1571$$

$$\mathrm{w}_{7,\mathrm{RR}}^{\text{best}}=\frac{2.571}{2.714+3.000+2.857+3.286+2.429+3.143+2.571}=\frac{2.571}{20}=0.1286$$

As we mentioned above, the sum of these weights should be equal to 1: $\sum_{j=1}^{7}\mathrm{w}_{j,\mathrm{RR}}^{\text{best}}=1$.

Similarly, the final ranks of criteria for the "worst case" are calculated using Table 6:

$$RR^{\text{worst}}(C_1)=\sum_{\mathrm{j}=1}^{7}\frac{(7-\mathrm{j}+1)\mathrm{r_j}}{7}=\sum_{\mathrm{j}=1}^{7}\frac{(8-\mathrm{i})\mathrm{r_i}}{7}=\frac{(8-1)*2}{7}+\frac{(8-4)*1}{7}+\frac{(8-7)*2}{7}=\frac{20}{7}=2.857$$

$$RR^{\text{worst}}(C_2)=\frac{(8-1)*1}{7}+\frac{(8-4)*1}{7}+\frac{(8-5)*1}{7}++\frac{(8-7)*2}{7}=\frac{16}{7}=2.286$$

$$RR^{\text{worst}}(C_3)=\frac{(8-3)*1}{7}+\frac{(8-5)*2}{7}+\frac{(8-6)*1}{7}+\frac{(8-7)*1}{7}=\frac{14}{7}=2.000$$

$$RR^{\text{worst}}(C_4)=\frac{(8-1)*1}{7}+\frac{(8-2)*1}{7}+\frac{(8-3)*1}{7}+\frac{(8-4)*2}{7}=\frac{26}{7}=3.714$$

$$RR^{\text{worst}}(C_5)=\frac{(8-1)*1}{7}+\frac{(8-3)*2}{7}+\frac{(8-5)*1}{7}+\frac{(8-6)*1}{7}=\frac{22}{7}=3.143$$

$$RR^{\text{worst}}(C_6)=\frac{(8-2)*2}{7}+\frac{(8-3)*1}{7}+\frac{(8-5)*1}{7}+\frac{(8-6)*1}{7}=\frac{22}{7}=3.143$$

$$RR^{\text{worst}}(C_7)=\frac{(8-2)*2}{7}+\frac{(8-4)*1}{7}+\frac{(8-6)*2}{7}=\frac{20}{7}=2.857$$

If considering the results obtained for the worst case according to the ranks in the calculation of weights, then the criteria weights will be as follows (Table 13).

$$\mathrm{w}_{1,\mathrm{RR}}^{\text{worst}}=\frac{2.857}{2.857+2.286+2.000+3.714+3.143+3.143+2.857}=\frac{2.857}{20}=0.1429$$

$$\mathrm{w}_{2,\mathrm{RR}}^{\mathrm{worst}} = \frac{2.286}{2.857 + 2.286 + 2.000 + 3.714 + 3.143 + 3.143 + 2.857} = \frac{2.286}{20} = 0.1143$$
$$\mathrm{w}_{3,\mathrm{RR}}^{\mathrm{worst}} = \frac{2.000}{2.857 + 2.286 + 2.000 + 3.714 + 3.143 + 3.143 + 2.857} = \frac{2.000}{20} = 0.1000$$
$$\mathrm{w}_{4,\mathrm{RR}}^{\mathrm{worst}} = \frac{3.714}{2.857 + 2.286 + 2.000 + 3.714 + 3.143 + 3.143 + 2.857} = \frac{3.714}{20} = 0.1857$$
$$\mathrm{w}_{5,\mathrm{RR}}^{\mathrm{worst}} = \frac{3.143}{2.857 + 2.286 + 2.000 + 3.714 + 3.143 + 3.143 + 2.857} = \frac{3.143}{20} = 0.1571$$
$$\mathrm{w}_{6,\mathrm{RR}}^{\mathrm{worst}} = \frac{3.143}{2.857 + 2.286 + 2.000 + 3.714 + 3.143 + 3.143 + 2.857} = \frac{3.143}{20} = 0.1571$$
$$\mathrm{w}_{7,\mathrm{RR}}^{\mathrm{worst}} = \frac{2.857}{2.857 + 2.286 + 2.000 + 3.714 + 3.143 + 3.143 + 2.857} = \frac{2.857}{20} = 0.1429$$

As we mentioned above, the sum of these weights should be equal to 1.

$$\sum_{j=1}^{7} \mathrm{w}_{\mathrm{j,RR}}^{\mathrm{worst}} = 1$$

As seen from Table 3, the most important criteria for DM1, DM2, DM3, DM4, DM5 decision makers are C_4, C_1, C_2, C_1 and C_3, respectively.

As seen from Table 4, the least important criteria for DM1, DM2, DM3, DM4, DM5 decision makers are C_4, C_5, C_1, C_2 and C_1, respectively.

Table 13. Weights of criteria assessment for "Worst case" and "Best case" based on final ranking (RR)

Criteria	Best	Worst	Average weights of the criteria
C_1	$\mathrm{w}_{1,\mathrm{RR}}^{\mathrm{best}} = 0.1357$	$\mathrm{w}_{1,\mathrm{RR}}^{\mathrm{worst}} = 0.1429$	$\mathrm{w}_{1,\mathrm{RR}}^{avg} = 0.1393$
C_2	$\mathrm{w}_{2,\mathrm{RR}}^{\mathrm{best}} = 0.1500$	$\mathrm{w}_{2,\mathrm{RR}}^{\mathrm{worst}} = 0.1143$	$\mathrm{w}_{2,\mathrm{RR}}^{avg} = 0.1321$
C_3	$\mathrm{w}_{3,\mathrm{RR}}^{\mathrm{best}} = 0.1429$	$\mathrm{w}_{3,\mathrm{RR}}^{\mathrm{worst}} = 0.1000$	$\mathrm{w}_{3,\mathrm{RR}}^{avg} = 0.1214$
C_4	$\mathrm{w}_{4,\mathrm{RR}}^{\mathrm{best}} = 0.1643$	$\mathrm{w}_{4,\mathrm{RR}}^{\mathrm{worst}} = 0.1857$	$\mathrm{w}_{4,\mathrm{RR}}^{avg} = 0.1750$
C_5	$\mathrm{w}_{5,\mathrm{RR}}^{\mathrm{best}} = 0.1214$	$\mathrm{w}_{5,\mathrm{RR}}^{\mathrm{worst}} = 0.1571$	$\mathrm{w}_{5,\mathrm{RR}}^{avg} = 0.1393$
C_6	$\mathrm{w}_{6,\mathrm{RR}}^{\mathrm{best}} = 0.1571$	$\mathrm{w}_{6,\mathrm{RR}}^{\mathrm{worst}} = 0.1571$	$\mathrm{w}_{6,\mathrm{RR}}^{avg} = 0.1571$
C_7	$\mathrm{w}_{7,\mathrm{RR}}^{\mathrm{best}} = 0.1286$	$\mathrm{w}_{7,\mathrm{RR}}^{\mathrm{worst}} = 0.1429$	$\mathrm{w}_{7,\mathrm{RR}}^{avg} = 0.1357$

Calculation f criteria weights for the "best" and "worst" cases: Let's calculate the weights of criteria with the help of the "best case" and "worst case" methods given in Section 7.2. For instance, let's calculate the best C_4 and the worst C_2 criteria determined by DM1 using the Eqs. (22) and (25) according to the weights of the criteria. The comparison ranks given in Tables 3 and 4 are used for this purpose.

First, let's calculate the weight of the "best" criterion C_4. If we apply Eq. (22) to Table 3, we obtain:

$$w_{7,\mathrm{PD}}^{\mathrm{best}} = \frac{1}{\frac{1}{3} + \frac{1}{7} + \frac{1}{2} + \frac{1}{1} + \frac{1}{6} + \frac{1}{5} + \frac{1}{4}} = 0.3857$$

Then, the weights for the "best case" for each criterion were calculated as follows:

$$w_{1,\mathrm{PD}}^{\mathrm{best}} = \frac{1}{3} * 0.3857 = 0.1286$$
$$w_{2,\mathrm{PD}}^{\mathrm{best}} = \frac{1}{7} * 0.3857 = 0.0551$$
$$w_{3,\mathrm{PD}}^{\mathrm{best}} = \frac{1}{2} * 0.3857 = 0.1928$$
$$w_{4,\mathrm{PD}}^{\mathrm{best}} = \frac{1}{1} * 0.3857 = 0.3857$$
$$w_{5,\mathrm{PD}}^{\mathrm{best}} = \frac{1}{6} * 0.3857 = 0.0643$$
$$w_{6,\mathrm{PD}}^{\mathrm{best}} = \frac{1}{5} * 0.3857 = 0.0771$$
$$w_{7,\mathrm{PD}}^{\mathrm{best}} = \frac{1}{4} * 0.3857 = 0.0964$$

It is clear that:

$$\sum_{j=1}^{7} \mathrm{w}_{j,\mathrm{PD}}^{\mathrm{best}} = 1$$

Similarly, applying the Eq. (25) to Table 4 we obtain:

$$w_7^{\mathrm{worst}} = 4 + 1 + 3 + 7 + 5 + 6 + 2 = 28$$

Then, the weights for the "worst case" for each criterion were calculated as follows:

$$w_{1,\mathrm{PD}}^{\mathrm{worst}} = \frac{4}{28} = 0.1429$$
$$w_{2,\mathrm{PD}}^{\mathrm{worst}} = \frac{1}{28} = 0.0357$$
$$w_{3,\mathrm{PD}}^{\mathrm{worst}} = \frac{3}{28} = 0.1071$$
$$w_{4,\mathrm{PD}}^{\mathrm{worst}} = \frac{7}{28} = 0.2500$$
$$w_{5,\mathrm{PD}}^{\mathrm{worst}} = \frac{5}{28} = 0.1786$$

$$w_{6,\text{PD}}^{\text{worst}} = \frac{6}{28} = 0.2143$$

$$w_{7,\text{PD}}^{\text{worst}} = \frac{2}{28} = 0.0714$$

$$\sum_{j=1}^{7} \text{w}_{\text{j,PD}}^{\text{worst}} = 1$$

We can calculate the weights of the criteria by performing similar calculations for the "best" and "worst" criteria identified by other experts. The obtained results are given in Table 14 (the "best case") and Table 15 (the "worst case").

The aggregated criteria weights from Tables 14 and 15 are given in Table 24.

Table 14. Weights of criteria for best case (Preference Degree)

Criteria	The weights of criteria for each decision-maker W_j^{best}					The average best-Worst weights of criteria w_j^{best}
	DM1	DM2	DM3	DM4	DM5	
C_1	0.1286	0.0496	0.3920	0.0551	0.3920	0.2034
C_2	0.0551	0.0567	0.0490	0.3857	0.0560	0.1205
C_3	0.1928	0.1324	0.1307	0.0771	0.0490	0.1164
C_4	0.3857	0.1986	0.0784	0.0643	0.1307	0.1715
C_5	0.0643	0.3972	0.1960	0.1286	0.0784	0.1729
C_6	0.0771	0.0662	0.0560	0.0964	0.1960	0.0983
C_7	0.0964	0.0993	0.0980	0.1928	0.0980	0.1169

Table 15. Weights of criteria for worst case (Preference Degree)

Criteria	The weights of criteria for each decision-maker W_j^{worst}					The average worst weights of criteria w_j^{worst}
	DM1	DM2	DM3	DM4	DM5	
C_1	0.1429	0.0345	0.2353	0.0357	0.2500	0.2036
C_2	0.0357	0.1379	0.0294	0.2500	0.0938	0.1113
C_3	0.1071	0.0690	0.1176	0.1786	0.0313	0.1020
C_4	0.2500	0.1724	0.1471	0.1429	0.2188	0.1793
C_5	0.1786	0.2759	0.0882	0.1071	0.1875	0.1300
C_6	0.2143	0.1034	0.1765	0.2143	0.0625	0.1422
C_7	0.0714	0.2069	0.2059	0.0714	0.1563	0.1316

Table 16. Weights of criteria for worst and the best case (Preference Degree)

Criteria	Best	Worst	Average weights of the criteria
C_1	$\text{w}_1^{\text{best}} = 0.2034$	$\text{w}_1^{\text{worst}} = 0.2036$	$\text{w}_1^{\text{bestworst}} = 0.2035$
C_2	$\text{w}_2^{\text{best}} = 0.1205$	$\text{w}_2^{\text{worst}} = 0.1113$	$\text{w}_2^{\text{bestworst}} = 0.1159$
C_3	$\text{w}_3^{\text{best}} = 0.1164$	$\text{w}_3^{\text{worst}} = 0.1020$	$\text{w}_3^{\text{bestworst}} = 0.1092$
C_4	$\text{w}_4^{\text{best}} = 0.1715$	$\text{w}_4^{\text{worst}} = 0.1793$	$\text{w}_4^{\text{bestworst}} = 0.1754$
C_5	$\text{w}_5^{\text{best}} = 0.1729$	$\text{w}_5^{\text{worst}} = 0.1300$	$\text{w}_5^{\text{bestworst}} = 0.1514$
C_6	$\text{w}_6^{\text{best}} = 0.0983$	$\text{w}_6^{\text{worst}} = 0.1422$	$\text{w}_6^{\text{bestworst}} = 0.1202$
C_7	$\text{w}_7^{\text{best}} = 0.1169$	$\text{w}_7^{\text{worst}} = 0.1316$	$\text{w}_7^{\text{bestworst}} = 0.1242$

Table 17. A weighted aggregated decision matrix for the "best case" (Preference Degree)

Alternatives	Criteria																				
	C1			**C2**			**C3**			**C4**			**C5**			**C6**			**C7**		
A_1	1.1402	1.3438	1.5475	0.4898	0.5566	0.6679	0.2856	0.3877	0.4897	0.7174	0.8967	1.0761	0.7798	0.9097	1.0397	0.4549	0.5402	0.6823	0.7106	0.8158	0.9474
A_2	0.8552	1.0181	1.2217	0.5566	0.6457	0.7570	0.4693	0.5509	0.6529	0.7533	0.8967	1.0761	0.4679	0.5458	0.6758	0.5117	0.5970	0.7392	0.4474	0.5527	0.6843
A_3	0.8145	1.0181	1.2217	0.7124	0.8238	0.9351	0.2040	0.2652	0.3673	0.6815	0.8250	1.0044	0.2859	0.3899	0.5198	0.5402	0.6539	0.7960	0.4474	0.5527	0.6843
A_4	0.7737	0.9366	1.1402	0.6457	0.7347	0.8460	0.4081	0.4693	0.5713	0.6815	0.8250	1.0044	0.3899	0.4939	0.6238	0.4833	0.5970	0.7392	0.6843	0.7632	0.8948
A_5	1.1402	1.3438	1.5475	0.4008	0.4675	0.5789	0.3060	0.4081	0.5101	0.6815	0.8250	1.0044	0.6238	0.7538	0.8837	0.4549	0.5970	0.7392	0.2895	0.3421	0.4737
A_6	0.5701	0.6923	0.8959	0.4898	0.6011	0.7124	0.4081	0.5101	0.6121	1.0044	1.1837	1.3631	0.4419	0.5458	0.6758	0.7108	0.8245	0.9666	0.1842	0.2369	0.3684
A_7	1.0181	1.1810	1.3846	0.3340	0.4230	0.5343	0.4897	0.5917	0.6937	0.4663	0.6098	0.7891	0.2079	0.2339	0.3639	0.7676	0.8813	1.0235	0.2369	0.2895	0.4211
A_8	0.4887	0.6108	0.8145	0.5343	0.6457	0.7570	0.5305	0.6325	0.7345	1.0044	1.1837	1.3631	0.3639	0.4419	0.5718	0.4265	0.4833	0.6255	0.5790	0.7106	0.8422
A_9	0.9773	1.1810	1.3846	0.5789	0.6902	0.8015	0.5305	0.5917	0.6937	0.2152	0.2511	0.4304	0.2599	0.3899	0.5198	1.0803	1.2225	1.3646	0.6316	0.7632	0.8948
A_{10}	0.8959	1.0995	1.3031	0.6011	0.6902	0.8015	0.4489	0.5509	0.6529	0.8967	1.0402	1.2196	0.4419	0.5458	0.6758	0.5402	0.5970	0.7392	0.4211	0.5527	0.6843
X^+	1.1402	1.3438	1.5475	0.7124	0.8238	0.9351	0.5305	0.6325	0.7345	1.0044	1.1837	1.3631	0.7798	0.9097	1.0397	1.0803	1.2225	1.3646	0.7106	0.8158	0.9474
X^-	0.4887	0.6108	0.8145	0.3340	0.4230	0.5343	0.2040	0.2652	0.3673	0.2152	0.2511	0.4304	0.2079	0.2339	0.3639	0.4265	0.4833	0.6255	0.1842	0.2369	0.3684

Table 18. A weighted aggregated decision matrix for the "worst case" (Preference Degree)

Alternatives	Criteria																				
	C1			**C2**			**C3**			**C4**			**C5**			**C6**			**C7**		
A_1	1.1393	1.3428	1.5462	0.5302	0.6025	0.7230	0.3259	0.4423	0.5587	0.6861	0.8576	1.0291	1.0373	1.2101	1.3830	0.3147	0.3737	0.4721	0.6313	0.7248	0.8417
A_2	0.8545	1.0172	1.2207	0.6025	0.6989	0.8194	0.5355	0.6286	0.7450	0.7204	0.8576	1.0291	0.6224	0.7261	0.8990	0.3540	0.4131	0.5114	0.3975	0.4910	0.6079
A_3	0.8138	1.0172	1.2207	0.7712	0.8917	1.0122	0.2328	0.3026	0.4191	0.6518	0.7890	0.9605	0.3803	0.5186	0.6915	0.3737	0.4524	0.5507	0.3975	0.4910	0.6079
A_4	0.7731	0.9359	1.1393	0.6989	0.7953	0.9158	0.4656	0.5355	0.6519	0.6518	0.7890	0.9605	0.5186	0.6569	0.8298	0.3344	0.4131	0.5114	0.6079	0.6781	0.7950
A_5	1.1393	1.3428	1.5462	0.4338	0.5061	0.6266	0.3492	0.4656	0.5820	0.6518	0.7890	0.9605	0.8298	1.0027	1.1756	0.3147	0.4131	0.5114	0.2572	0.3040	0.4209
A_6	0.5697	0.6917	0.8952	0.5302	0.6507	0.7712	0.4656	0.5820	0.6984	0.9605	1.1320	1.3035	0.5878	0.7261	0.8990	0.4917	0.5704	0.6688	0.1637	0.2104	0.3273
A_7	1.0172	1.1800	1.3835	0.3615	0.4579	0.5784	0.5587	0.6751	0.7915	0.4459	0.5832	0.7547	0.2766	0.3112	0.4841	0.5311	0.6097	0.7081	0.2104	0.2572	0.3741
A_8	0.4883	0.6103	0.8138	0.5784	0.6989	0.8194	0.6053	0.7217	0.8381	0.9605	1.1320	1.3035	0.4841	0.5878	0.7607	0.2950	0.3344	0.4327	0.5144	0.6313	0.7482
A_9	0.9766	1.1800	1.3835	0.6266	0.7471	0.8676	0.6053	0.6751	0.7915	0.2058	0.2401	0.4116	0.3458	0.5186	0.6915	0.7474	0.8458	0.9441	0.5612	0.6781	0.7950
A_{10}	0.8952	1.0986	1.3021	0.6507	0.7471	0.8676	0.5122	0.6286	0.7450	0.8576	0.9948	1.1663	0.5878	0.7261	0.8990	0.3737	0.4131	0.5114	0.3741	0.4910	0.6079
X^+	1.1393	1.3428	1.5462	0.7712	0.8917	1.0122	0.6053	0.7217	0.8381	0.9605	1.1320	1.3035	1.0373	1.2101	1.3830	0.7474	0.8458	0.9441	0.6313	0.7248	0.8417
X^-	0.4883	0.6103	0.8138	0.3615	0.4579	0.5784	0.2328	0.3026	0.4191	0.2058	0.2401	0.4116	0.2766	0.3112	0.4841	0.2950	0.3344	0.4327	0.1637	0.2104	0.3273

Table 19. A weighted aggregated decision matrix for calculating the average value (Preference Degree)

Alternatives	Criteria																				
	C1			C2			C3			C4			C5			C6			C7		
A_1	1.1398	1.3433	1.5468	0.5100	0.5796	0.6955	0.3058	0.4150	0.5242	0.7017	0.8772	1.0526	0.9085	1.0599	1.2114	0.3848	0.4569	0.5772	0.6709	0.7703	0.8946
A_2	0.8548	1.0177	1.2212	0.5796	0.6723	0.7882	0.5024	0.5897	0.6989	0.7368	0.8772	1.0526	0.5451	0.6360	0.7874	0.4329	0.5050	0.6253	0.4224	0.5218	0.6461
A_3	0.8141	1.0177	1.2212	0.7418	0.8577	0.9736	0.2184	0.2839	0.3932	0.6666	0.8070	0.9824	0.3331	0.4543	0.6057	0.4569	0.5531	0.6734	0.4224	0.5218	0.6461
A_4	0.7734	0.9362	1.1398	0.6723	0.7650	0.8809	0.4368	0.5024	0.6116	0.6666	0.8070	0.9824	0.4543	0.5754	0.7268	0.4088	0.5050	0.6253	0.6461	0.7206	0.8449
A_5	1.1398	1.3433	1.5468	0.4173	0.4868	0.6027	0.3276	0.4368	0.5460	0.6666	0.8070	0.9824	0.7268	0.8782	1.0296	0.3848	0.5050	0.6253	0.2733	0.3230	0.4473
A_6	0.5699	0.6920	0.8955	0.5100	0.6259	0.7418	0.4368	0.5460	0.6553	0.9824	1.1579	1.3333	0.5148	0.6360	0.7874	0.6012	0.6974	0.8177	0.1739	0.2236	0.3479
A_7	1.0177	1.1805	1.3840	0.3477	0.4405	0.5564	0.5242	0.6334	0.7426	0.4561	0.5965	0.7719	0.2423	0.2726	0.4240	0.6493	0.7455	0.8658	0.2236	0.2733	0.3976
A_8	0.4885	0.6106	0.8141	0.5564	0.6723	0.7882	0.5679	0.6771	0.7863	0.9824	1.1579	1.3333	0.4240	0.5148	0.6662	0.3607	0.4088	0.5291	0.5467	0.6709	0.7952
A_9	0.9770	1.1805	1.3840	0.6027	0.7186	0.8346	0.5679	0.6334	0.7426	0.2105	0.2456	0.4210	0.3028	0.4543	0.6057	0.9139	1.0341	1.1544	0.5964	0.7206	0.8449
A_{10}	0.8955	1.0991	1.3026	0.6259	0.7186	0.8346	0.4805	0.5897	0.6989	0.8772	1.0175	1.1929	0.5148	0.6360	0.7874	0.4569	0.5050	0.6253	0.3976	0.5218	0.6461
X^+	1.1398	1.3433	1.5468	0.7418	0.8577	0.9736	0.5679	0.6771	0.7863	0.9824	1.1579	1.3333	0.9085	1.0599	1.2114	0.9139	1.0341	1.1544	0.6709	0.7703	0.8946
X^-	0.4885	0.6106	0.8141	0.3477	0.4405	0.5564	0.2184	0.2839	0.3932	0.2105	0.2456	0.4210	0.2423	0.2726	0.4240	0.3607	0.4088	0.5291	0.1739	0.2236	0.3479

Table 20. A weighted aggregated decision matrix for the "best case" (Result Rank)

Alternatives	Criteria																				
	C1			C2			C3			C4			C5			C6			C7		
A_1	0.8000	0.9429	1.0857	0.5029	0.5714	0.6857	0.2800	0.3800	0.4800	0.7429	0.9286	1.1143	0.9429	1.1000	1.2571	0.5029	0.5971	0.7543	0.7714	0.8857	1.0286
A_2	0.6000	0.7143	0.8571	0.5714	0.6629	0.7771	0.4600	0.5400	0.6400	0.7800	0.9286	1.1143	0.5657	0.6600	0.8171	0.5657	0.6600	0.8171	0.4857	0.6000	0.7429
A_3	0.5714	0.7143	0.8571	0.7314	0.8457	0.9600	0.2000	0.2600	0.3600	0.7057	0.8543	1.0400	0.3457	0.4714	0.6286	0.5971	0.7229	0.8800	0.4857	0.6000	0.7429
A_4	0.5429	0.6571	0.8000	0.6629	0.7543	0.8686	0.4000	0.4600	0.5600	0.7057	0.8543	1.0400	0.4714	0.5971	0.7543	0.5343	0.6600	0.8171	0.7429	0.8286	0.9714
A_5	0.8000	0.9429	1.0857	0.4114	0.4800	0.5943	0.3000	0.4000	0.5000	0.7057	0.8543	1.0400	0.7543	0.9114	1.0686	0.5029	0.6600	0.8171	0.3143	0.3714	0.5143
A_6	0.4000	0.4857	0.6286	0.5029	0.6171	0.7314	0.4000	0.5000	0.6000	1.0400	1.2257	1.4114	0.5343	0.6600	0.8171	0.7857	0.9114	1.0686	0.2000	0.2571	0.4000
A_7	0.7143	0.8286	0.9714	0.3429	0.4343	0.5486	0.4800	0.5800	0.6800	0.4829	0.6314	0.8171	0.2514	0.2829	0.4400	0.8486	0.9743	1.1314	0.2571	0.3143	0.4571
A_8	0.3429	0.4286	0.5714	0.5486	0.6629	0.7771	0.5200	0.6200	0.7200	1.0400	1.2257	1.4114	0.4400	0.5343	0.6914	0.4714	0.5343	0.6914	0.6286	0.7714	0.9143
A_9	0.6857	0.8286	0.9714	0.5943	0.7086	0.8229	0.5200	0.5800	0.6800	0.2229	0.2600	0.4457	0.3143	0.4714	0.6286	1.1943	1.3514	1.5086	0.6857	0.8286	0.9714
A_{10}	0.6286	0.7714	0.9143	0.6171	0.7086	0.8229	0.4400	0.5400	0.6400	0.9286	1.0771	1.2629	0.5343	0.6600	0.8171	0.5971	0.6600	0.8171	0.4571	0.6000	0.7429
X^+	0.8000	0.9429	1.0857	0.7314	0.8457	0.9600	0.5200	0.6200	0.7200	1.0400	1.2257	1.4114	0.9429	1.1000	1.2571	1.1943	1.3514	1.5086	0.7714	0.8857	1.0286
X^-	0.3429	0.4286	0.5714	0.3429	0.4343	0.5486	0.2000	0.2600	0.3600	0.2229	0.2600	0.4457	0.2514	0.2829	0.4400	0.4714	0.5343	0.6914	0.2000	0.2571	0.4000

Table 21. A weighted aggregated decision matrix for the "worst case" (Result Rank)

Alternatives	Criteria																				
	C1			C2			C3			C4			C5			C6			C7		
A_1	0.7600	0.8957	1.0314	0.6600	0.7500	0.9000	0.4000	0.5429	0.6857	0.6571	0.8214	0.9857	0.7286	0.8500	0.9714	0.5029	0.5971	0.7543	0.6943	0.7971	0.9257
A_2	0.5700	0.6786	0.8143	0.7500	0.8700	1.0200	0.6571	0.7714	0.9143	0.6900	0.8214	0.9857	0.4371	0.5100	0.6314	0.5657	0.6600	0.8171	0.4371	0.5400	0.6686
A_3	0.5429	0.6786	0.8143	0.9600	1.1100	1.2600	0.2857	0.3714	0.5143	0.6243	0.7557	0.9200	0.2671	0.3643	0.4857	0.5971	0.7229	0.8800	0.4371	0.5400	0.6686
A_4	0.5157	0.6243	0.7600	0.8700	0.9900	1.1400	0.5714	0.6571	0.8000	0.6243	0.7557	0.9200	0.3643	0.4614	0.5829	0.5343	0.6600	0.8171	0.6686	0.7457	0.8743
A_5	0.7600	0.8957	1.0314	0.5400	0.6300	0.7800	0.4286	0.5714	0.7143	0.6243	0.7557	0.9200	0.5829	0.7043	0.8257	0.5029	0.6600	0.8171	0.2829	0.3343	0.4629
A_6	0.3800	0.4614	0.5971	0.6600	0.8100	0.9600	0.5714	0.7143	0.8571	0.9200	1.0843	1.2486	0.4129	0.5100	0.6314	0.7857	0.9114	1.0686	0.1800	0.2314	0.3600
A_7	0.6786	0.7871	0.9229	0.4500	0.5700	0.7200	0.6857	0.8286	0.9714	0.4271	0.5586	0.7229	0.1943	0.2186	0.3400	0.8486	0.9743	1.1314	0.2314	0.2829	0.4114
A_8	0.3257	0.4071	0.5429	0.7200	0.8700	1.0200	0.7429	0.8857	1.0286	0.9200	1.0843	1.2486	0.3400	0.4129	0.5343	0.4714	0.5343	0.6914	0.5657	0.6943	0.8229
A_9	0.6514	0.7871	0.9229	0.7800	0.9300	1.0800	0.7429	0.8286	0.9714	0.1971	0.2300	0.3943	0.2429	0.3643	0.4857	1.1943	1.3514	1.5086	0.6171	0.7457	0.8743
A_{10}	0.5971	0.7329	0.8686	0.8100	0.9300	1.0800	0.6286	0.7714	0.9143	0.8214	0.9529	1.1171	0.4129	0.5100	0.6314	0.5971	0.6600	0.8171	0.4114	0.5400	0.6686
X^+	0.7600	0.8957	1.0314	0.9600	1.1100	1.2600	0.7429	0.8857	1.0286	0.9200	1.0843	1.2486	0.7286	0.8500	0.9714	1.1943	1.3514	1.5086	0.6943	0.7971	0.9257
X^-	0.3257	0.4071	0.5429	0.4500	0.5700	0.7200	0.2857	0.3714	0.5143	0.1971	0.2300	0.3943	0.1943	0.2186	0.3400	0.4714	0.5343	0.6914	0.1800	0.2314	0.3600

Table 22. A weighted aggregated decision matrix for calculating the average value (Result Rank)

Alternatives	Criteria																				
	C1			C2			C3			C4			C5			C6			C7		
A_1	0.7800	0.9193	1.0586	0.5814	0.6607	0.7929	0.3400	0.4614	0.5829	0.7000	0.8750	1.0500	0.8357	0.9750	1.1143	0.5029	0.5971	0.7543	0.7329	0.8414	0.9771
A_2	0.5850	0.6964	0.8357	0.6607	0.7664	0.8986	0.5586	0.6557	0.7771	0.7350	0.8750	1.0500	0.5014	0.5850	0.7243	0.5657	0.6600	0.8171	0.4614	0.5700	0.7057
A_3	0.5571	0.6964	0.8357	0.8457	0.9779	1.1100	0.2429	0.3157	0.4371	0.6650	0.8050	0.9800	0.3064	0.4179	0.5571	0.5971	0.7229	0.8800	0.4614	0.5700	0.7057
A_4	0.5293	0.6407	0.7800	0.7664	0.8721	1.0043	0.4857	0.5586	0.6800	0.6650	0.8050	0.9800	0.4179	0.5293	0.6686	0.5343	0.6600	0.8171	0.7057	0.7871	0.9229
A_5	0.7800	0.9193	1.0586	0.4757	0.5550	0.6871	0.3643	0.4857	0.6071	0.6650	0.8050	0.9800	0.6686	0.8079	0.9471	0.5029	0.6600	0.8171	0.2986	0.3529	0.4886
A_6	0.3900	0.4736	0.6129	0.5814	0.7136	0.8457	0.4857	0.6071	0.7286	0.9800	1.1550	1.3300	0.4736	0.5850	0.7243	0.7857	0.9114	1.0686	0.1900	0.2443	0.3800
A_7	0.6964	0.8079	0.9471	0.3964	0.5021	0.6343	0.5829	0.7043	0.8257	0.4550	0.5950	0.7700	0.2229	0.2507	0.3900	0.8486	0.9743	1.1314	0.2443	0.2986	0.4343
A_8	0.3343	0.4179	0.5571	0.6343	0.7664	0.8986	0.6314	0.7529	0.8743	0.9800	1.1550	1.3300	0.3900	0.4736	0.6129	0.4714	0.5343	0.6914	0.5971	0.7329	0.8686
A_9	0.6686	0.8079	0.9471	0.6871	0.8193	0.9514	0.6314	0.7043	0.8257	0.2100	0.2450	0.4200	0.2786	0.4179	0.5571	1.1943	1.3514	1.5086	0.6514	0.7871	0.9229
A_{10}	0.6129	0.7521	0.8914	0.7136	0.8193	0.9514	0.5343	0.6557	0.7771	0.8750	1.0150	1.1900	0.4736	0.5850	0.7243	0.5971	0.6600	0.8171	0.4343	0.5700	0.7057
X^+	0.7800	0.9193	1.0586	0.8457	0.9779	1.1100	0.6314	0.7529	0.8743	0.9800	1.1550	1.3300	0.8357	0.9750	1.1143	1.1943	1.3514	1.5086	0.7329	0.8414	0.9771
X^-	0.3343	0.4179	0.5571	0.3964	0.5021	0.6343	0.2429	0.3157	0.4371	0.2100	0.2450	0.4200	0.2229	0.2507	0.3900	0.4714	0.5343	0.6914	0.1900	0.2443	0.3800

Applying Eq.s (22) and (24), we obtain the following weights for criteria from Table 21.

Table 23. Defuzzified decision matrices (Preference Degree**)**

Alternatives	Criteria								
	Best			Worst			Average weights of the criteria		
A_1	4.5783	5.4506	6.4506	4.6648	5.5539	6.5539	4.6215	5.5022	6.5022
A_2	4.0613	4.8069	5.8069	4.0867	4.8325	5.8325	4.0740	4.8197	5.8197
A_3	3.6860	4.5285	5.5285	3.6211	4.4626	5.4626	3.6535	4.4956	5.4956
A_4	4.0664	4.8197	5.8197	4.0503	4.8036	5.8036	4.0584	4.8117	5.8117
A_5	3.8968	4.7374	5.7374	3.9758	4.8232	5.8232	3.9363	4.7803	5.7803
A_6	3.8092	4.5944	5.5944	3.7692	4.5634	5.5634	3.7892	4.5789	5.5789
A_7	3.5204	4.2102	5.2102	3.4015	4.0743	5.0743	3.4610	4.1423	5.1423
A_8	3.9272	4.7085	5.7085	3.9260	4.7164	5.7164	3.9266	4.7124	5.7124
A_9	4.2738	5.0895	6.0895	4.0686	4.8848	5.8848	4.1712	4.9872	5.9872
A_{10}	4.2458	5.0764	6.0764	4.2512	5.0993	6.0993	4.2485	5.0878	6.0878
X^+	4.5783	5.4506	6.4506	4.6648	5.5539	6.5539	4.6215	5.5022	6.5022
X^-	3.5204	4.2102	5.2102	3.4015	4.0743	5.0743	3.4610	4.1423	5.1423

Table 24. Defuzzified decision matrices (Result Rank**)**

Alternatives	Criteria								
	Best			Worst			Average weights of the criteria		
A_1	4.5429	5.4057	6.4057	4.4029	5.2543	6.2543	4.4729	5.3300	6.3300
A_2	4.0286	4.7657	5.7657	4.1071	4.8514	5.8514	4.0679	4.8086	5.8086
A_3	3.6371	4.4686	5.4686	3.7143	4.5429	5.5429	3.6757	4.5057	5.5057
A_4	4.0600	4.8114	5.8114	4.1486	4.8943	5.8943	4.1043	4.8529	5.8529
A_5	3.7886	4.6200	5.6200	3.7214	4.5514	5.5514	3.7550	4.5857	5.5857
A_6	3.8629	4.6571	5.6571	3.9100	4.7229	5.7229	3.8864	4.6900	5.6900
A_7	3.3771	4.0457	5.0457	3.5157	4.2200	5.2200	3.4464	4.1329	5.1329
A_8	3.9914	4.7771	5.7771	4.0857	4.8886	5.8886	4.0386	4.8329	5.8329
A_9	4.2171	5.0286	6.0286	4.4257	5.2371	6.2371	4.3214	5.1329	6.1329
A_{10}	4.2029	5.0171	6.0171	4.2786	5.0971	6.0971	4.2407	5.0571	6.0571
X^+	4.5429	5.4057	6.4057	4.4257	5.2543	6.2543	4.4729	5.3300	6.3300
X^-	3.3771	4.0457	5.0457	3.5157	4.2200	5.2200	3.4464	4.1329	5.1329

Table 25. Ranking of alternatives based on TOPSIS values for the "best case" and the "worst case" (Result Rank**)**

Alternatives	Best				Worst				Average value			
	D_i^+	D_i^-	V_i	Rank	D_i^+	D_i^-	V_i	Rank	D_i^+	D_i^-	V_i	Rank
A_1	0.0000	1.1827	1,0000	1	0.0000	1.4111	1,0000	1	0.0000	1.2969	1,0000	1
A_2	0.6044	0.5787	0,4891	6	0.6770	0.7346	0,5204	5	0.6407	0.6567	0,5061	5
A_3	0.9123	0.2769	0,2329	9	1.0756	0.3414	0,2409	9	0.9940	0.3091	0,2372	9
A_4	0.5939	0.5891	0,4980	5	0.7079	0.7035	0,4985	6	0.6509	0.6463	0,4982	6
A_5	0.7028	0.4822	0,4069	3	0.7171	0.6955	0,4924	2	0.7099	0.5888	0,4534	3
A_6	0.8282	0.3552	0,3002	7	0.9599	0.4522	0,3202	7	0.8941	0.4037	0,3111	7
A_7	1.1827	0.0000	0,0000	10	1.4111	0.0000	0,0000	10	1.2969	0.0000	0,0000	10
A_8	0.7131	0.4697	0,3971	8	0.8059	0.6054	0,4290	8	0.7595	0.5376	0,4145	8
A_9	0.3433	0.8394	0,7098	2	0.6457	0.7657	0,5425	3	0.4944	0.8025	0,6188	2
A_{10}	0.3609	0.8219	0,6949	4	0.4413	0.9700	0,6873	4	0.4011	0.8960	0,6908	4

Table 26. Ranking of alternatives based on TOPSIS values for the "best case" and the "worst case" (Preference Degree)

Alternatives	Best				Worst				Average value			
	D_i^+	D_i^-	V_i	Rank	D_i^+	D_i^-	V_i	Rank	D_i^+	D_i^-	V_i	Rank
A_1	0.0000	1.2985	1,0000	1	0.0000	0.9877	0,9868	2	0.0000	1.1431	1,0000	2
A_2	0.6010	0.6979	0,5373	7	0.3637	0.6184	0,6213	8	0.4857	0.6581	0,5754	8
A_3	0.9268	0.3765	0,2889	9	0.6982	0.2875	0,2878	9	0.8153	0.3320	0,2893	9
A_4	0.5596	0.7391	0,5691	5	0.3215	0.6608	0,6638	5	0.4439	0.6999	0,6119	5
A_5	0.7754	0.5256	0,4040	3	0.6901	0.2955	0,2959	4	0.7356	0.4104	0,3581	4
A_6	0.7264	0.5726	0,4408	6	0.5130	0.4695	0,4715	7	0.6227	0.5210	0,4556	6
A_7	1.2985	0.0000	0,0000	10	0.9814	0.0000	0,0000	10	1.1431	0.0000	0,0000	10
A_8	0.6040	0.6946	0,5349	8	0.3442	0.6374	0,6408	6	0.4771	0.6660	0,5826	7
A_9	0.3608	0.9377	0,7221	2	0.0008	0.9827	0,9860	1	0.1832	0.9601	0,8398	1
A_{10}	0.3731	0.9254	0,7127	4	0.1407	0.8408	0,8453	3	0.2600	0.8831	0,7725	3

Analysis and discussion:

1) "Best case" method

TOPSIS (PD)

$$\frac{0.7221-1.0000}{0.0000-1.0000}=0.2779\geq\frac{1}{10-1}$$

TOPSIS (RR)

$$\frac{0.7098-1.0000}{0.0000-1.0000}=\mathbf{0.2902}\geq\frac{1}{10-1}$$

2) "Worst case" method

TOPSIS (PD)

$$\frac{0.9860-0.9868}{0.0000-0.9868}=0.0008<\frac{1}{10-1}$$

TOPSIS (RR)

$$\frac{0.6873-1.0000}{0.0000-1.0000}=\mathbf{0.3127}\geq\frac{1}{10-1}$$

3) (Best+Worst)/2

TOPSIS (PD)

$$\frac{0.8398-1.0000}{0.0000-1.0000}=0.1602\geq\frac{1}{10-1}$$

TOPSIS (RR)

$$\frac{0.6908-1.0000}{0.0000-1.0000}=\mathbf{0.3092}\geq\frac{1}{10-1}$$

Using Eqs. (4)-(9) for $\lambda = 0.3, 0.4, 0.6, 0.7$ values, S, R and Q values were calculated. Then, using the Eq. (9), their fuzzy values are defuzzified and listed in Tables 26-30.

Table 27. Ranking alternatives based on their Q^V values for $\lambda = 0.3$, $\lambda = 0.4$, $\lambda = 0.6$ and $\lambda = 0.7$ (PD)

Alternatives	$\lambda = 0.3$			$\lambda = 0.4$			$\lambda = 0.6$			$\lambda = 0.7$		
	Q^V best	Q^V Worst	Q^V average	Q^V best	Q^V Worst	Q^V average	Q^V best	Q^V Worst	Q^V average	Q^V best	Q^V Worst	Q^V average
A_1	0.7000	0.7000	0.7000	0.6000	0.6000	0.6000	0.4000	0.4000	0.4000	0.3000	0.3000	0.3000
A_2	0.5151	0.5494	0.5305	0.5076	0.5239	0.5154	0.4927	0.4728	0.4853	0.4853	0.4472	0.4702
A_3	0.4171	0.4172	0.4173	0.4595	0.4592	0.4596	0.5442	0.5433	0.5441	0.5866	0.5853	0.5864
A_4	0.5278	0.5666	0.5451	0.5139	0.5324	0.5227	0.4863	0.4641	0.4779	0.4725	0.4300	0.4555
A_5	0.4625	0.4204	0.4444	0.4817	0.4608	0.4728	0.5202	0.5416	0.5297	0.5394	0.5820	0.5582
A_6	0.4765	0.4896	0.4825	0.4884	0.4943	0.4914	0.5121	0.5038	0.5092	0.5239	0.5085	0.5180
A_7	0.3000	0.3000	0.3000	0.4000	0.4000	0.4000	0.6000	0.6000	0.6000	0.7000	0.7000	0.7000
A_8	0.5140	0.5570	0.5331	0.5070	0.5275	0.5165	0.4930	0.4686	0.4835	0.4861	0.4391	0.4670
A_9	0.5889	0.6967	0.6361	0.5444	0.5973	0.5681	0.4556	0.3985	0.4321	0.4112	0.2991	0.3642
A_{10}	0.5851	0.6389	0.6090	0.5425	0.5681	0.5545	0.4575	0.4265	0.4455	0.4149	0.3557	0.3910

Table 28. Ranking alternatives based on their Q^V values for $\lambda = 0.3, \lambda = 0.4, \lambda = 0.6$ and $\lambda = 0.7$ (RR)

Alternatives	$\lambda = 0.3$			$\lambda = 0.4$			$\lambda = 0.6$			$\lambda = 0.7$		
	Q^V best	Q^V Worst	Q^V average	Q^V best	Q^V Worst	Q^V average	Q^V best	Q^V Worst	Q^V average	Q^V best	Q^V Worst	Q^V average
A_1	0.7000	0.7000	0.7000	0.6000	0.6000	0.6000	0.4000	0.4000	0.4000	0.3000	0.3000	0.3000
A_2	0.4958	0.5083	0.5026	0.4980	0.5043	0.5014	0.5024	0.4961	0.4989	0.5045	0.4920	0.4977
A_3	0.3953	0.3980	0.3968	0.4490	0.4501	0.4496	0.5565	0.5541	0.5552	0.6102	0.6062	0.6080
A_4	0.4993	0.4995	0.4994	0.4997	0.4998	0.4997	0.5005	0.5004	0.5005	0.5009	0.5007	0.5008
A_5	0.4637	0.4975	0.4820	0.4823	0.4990	0.4914	0.5196	0.5020	0.5100	0.5383	0.5036	0.5194
A_6	0.4203	0.4284	0.4247	0.4603	0.4644	0.4625	0.5403	0.5363	0.5381	0.5803	0.5723	0.5760
A_7	0.3000	0.3000	0.3000	0.4000	0.4000	0.4000	0.6000	0.6000	0.6000	0.7000	0.7000	0.7000
A_8	0.4589	0.4717	0.4658	0.4795	0.4859	0.4830	0.5206	0.5143	0.5172	0.5412	0.5285	0.5343
A_9	0.5839	0.5171	0.5475	0.5420	0.5086	0.5238	0.4581	0.4916	0.4763	0.4161	0.4831	0.4525
A_{10}	0.5780	0.5750	0.5764	0.5390	0.5376	0.5382	0.4611	0.4626	0.4619	0.4221	0.4252	0.4238

Verification of the accepted preference condition:

Best $\lambda = 0.3$

VIKOR (PD)

$$\frac{0.4171 - 0.3000}{0.7000 - 0.3000} = \mathbf{0.2927} \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.3953 - 0.3000}{0.7000 - 0.3000} = 0.2353 \geq \frac{1}{10 - 1}$$

Worst

VIKOR (PD)

$$\frac{0.4172 - 0.3000}{0.7000 - 0.3000} = \mathbf{0.2929} \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.3980 - 0.3000}{0.7000 - 0.3000} = 0.2451 \geq \frac{1}{10 - 1}$$

(Best+Worst)/2

VIKOR (PD)

$$\frac{0.4173 - 0.3000}{0.7000 - 0.3000} = \mathbf{0.2932} \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.3968 - 0.3000}{0.7000 - 0.3000} = 0.2420 \geq \frac{1}{10 - 1}$$

Best $\lambda = 0.4$

VIKOR (PD)

$$\frac{0.4595 - 0.4000}{0.6000 - 0.4000} = \mathbf{0.2975} \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4490 - 0.4000}{0.6000 - 0.4000} = 0.2450 \geq \frac{1}{10 - 1}$$

Worst

VIKOR (PD)

$$\frac{0.4608 - 0.4000}{0.6000 - 0.4000} = \mathbf{0.3041} \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4501 - 0.4000}{0.6000 - 0.4000} = 0.2504 \geq \frac{1}{10 - 1}$$

(Best+Worst)/2

VIKOR (PD)

$$\frac{0.4596 - 0.4000}{0.6000 - 0.4000} = \mathbf{0.2978} \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4496 - 0.4000}{0.6000 - 0.4000} = 0.2479 \geq \frac{1}{10 - 1}$$

Best $\lambda = 0.6$

VIKOR (PD)

$$\frac{0.4556 - 0.4000}{0.6000 - 0.4000} = 0.2780 \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4581 - 0.4000}{0.6000 - 0.4000} = \mathbf{0.2905} \geq \frac{1}{10 - 1}$$

Worst

VIKOR (PD)

$$\frac{0.4000 - 0.3985}{0.6000 - 0.4000} = 0.0007 < \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4626 - 0.4000}{0.6000 - 0.4000} = \mathbf{0.3130} \geq \frac{1}{10 - 1}$$

(Best+Worst)/2

VIKOR (PD)

$$\frac{0.4321 - 0.4000}{0.6000 - 0.4000} = 0.1605 \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4619 - 0.4000}{0.6000 - 0.4000} = \mathbf{0.3095} \geq \frac{1}{10 - 1}$$

Best $\lambda = 0.7$

VIKOR (PD)

$$\frac{0.4112 - 0.3000}{0.7000 - 0.3000} = 0.2780 \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4161 - 0.3000}{0.7000 - 0.3000} = \mathbf{0.2902} \geq \frac{1}{10 - 1}$$

Worst

VIKOR (PD)

$$\frac{0.3000 - 0.2991}{0.7000 - 0.3000} = 0.0022 < \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4252 - 0.3000}{0.7000 - 0.3000} = \mathbf{0.3130} \geq \frac{1}{10 - 1}$$

(Best+Worst)/2

VIKOR (PD)

$$\frac{0.3642 - 0.3000}{0.7000 - 0.3000} = 0.1605 \geq \frac{1}{10 - 1}$$

VIKOR (RR)

$$\frac{0.4238 - 0.3000}{0.7000 - 0.3000} = \mathbf{0.3098} \geq \frac{1}{10 - 1}$$

Calculation of the TOPSIS method.

First, the values calculated for each alternative according to Table 20 are summed and divided by the number of experts. Then, the obtained values are multiplied by the weight of criteria given in Table 22, respectively.

Table 31 lists the aggregated values of the alternatives to the relevant criteria calculated according to the Eq. (9).

Table 29. Aggregated values of alternatives according to relevant criteria based on preference degree

	Best			**Worst**			**(Best+worst)/2**		
A_1	4.5783	5.4506	6.4506	4.6648	5.5539	6.5539	4.6215	5.5022	6.5022
A_2	4.0613	4.8069	5.8069	4.0867	4.8325	5.8325	4.0740	4.8197	5.8197
A_3	3.6860	4.5285	5.5285	3.6211	4.4626	5.4626	3.6535	4.4956	5.4956
A_4	4.0664	4.8197	5.8197	4.0503	4.8036	5.8036	4.0584	4.8117	5.8117
A_5	3.8968	4.7374	5.7374	3.9758	4.8232	5.8232	3.9363	4.7803	5.7803
A_6	3.8092	4.5944	5.5944	3.7692	4.5634	5.5634	3.7892	4.5789	5.5789
A_7	3.5204	4.2102	5.2102	3.4015	4.0743	5.0743	3.4610	4.1423	5.1423
A_8	3.9272	4.7085	5.7085	3.9260	4.7164	5.7164	3.9266	4.7124	5.7124
A_9	4.2738	5.0895	6.0895	4.0686	4.8848	5.8848	4.1712	4.9872	5.9872
A_{10}	4.2458	5.0764	6.0764	4.2512	5.0993	6.0993	4.2485	5.0878	6.0878
max	4.5783	5.4506	6.4506	4.6648	5.5539	6.5539	4.6215	5.5022	6.5022
min	3.5204	4.2102	5.2102	3.4015	4.0743	5.0743	3.4610	4.1423	5.1423

Based on Eq.s (12) and (13), the Euclidean distance of the best and worst ideal solution was calculated and shown in Table 32. Max and min values are taken from Table 31.

Table 30. The best and the worst ideal solution matrix based on preference degree

	Best			Worst			(Best+worst)/2		
	S+	S-	V	S+	S-	V	S+	S-	V
A_1	0.0000	1.1827	1.0000	0.0000	1.4111	1.0000	0.0000	1.2969	1.0000
A_2	0.6044	0.5787	0.4891	0.6770	0.7346	0.5204	0.6407	0.6567	0.5061
A_3	0.9123	0.2769	0.2329	1.0756	0.3414	0.2409	0.9940	0.3091	0.2372
A_4	0.5939	0.5891	0.4980	0.7079	0.7035	0.4985	0.6509	0.6463	0.4982
A_5	0.7028	0.4822	0.4069	0.7171	0.6955	0.4924	0.7099	0.5888	0.4534
A_6	0.8282	0.3552	0.3002	0.9599	0.4522	0.3202	0.8941	0.4037	0.3111
A_7	1.1827	0.0000	0.0000	1.4111	0.0000	0.0000	1.2969	0.0000	0.0000
A_8	0.7131	0.4697	0.3971	0.8059	0.6054	0.4290	0.7595	0.5376	0.4145
A_9	0.3433	0.8394	0.7098	0.6457	0.7657	0.5425	0.4944	0.8025	0.6188
A_{10}	0.3609	0.8219	0.6949	0.4413	0.9700	0.6873	0.4011	0.8960	0.6908

Table 31. Aggregated values of the alternatives according to the relevant criteria based on the ranking

	Best			Worst			(Best+worst)/2		
A_1	4.5429	5.4057	6.4057	4.4029	5.2543	6.2543	4.4729	5.3300	6.3300
A_2	4.0286	4.7657	5.7657	4.1071	4.8514	5.8514	4.0679	4.8086	5.8086
A_3	3.6371	4.4686	5.4686	3.7143	4.5429	5.5429	3.6757	4.5057	5.5057
A_4	4.0600	4.8114	5.8114	4.1486	4.8943	5.8943	4.1043	4.8529	5.8529
A_5	3.7886	4.6200	5.6200	3.7214	4.5514	5.5514	3.7550	4.5857	5.5857
A_6	3.8629	4.6571	5.6571	3.9100	4.7229	5.7229	3.8864	4.6900	5.6900
A_7	3.3771	4.0457	5.0457	3.5157	4.2200	5.2200	3.4464	4.1329	5.1329
A_8	3.9914	4.7771	5.7771	4.0857	4.8886	5.8886	4.0386	4.8329	5.8329
A_9	4.2171	5.0286	6.0286	4.4257	5.2371	6.2371	4.3214	5.1329	6.1329
A_{10}	4.2029	5.0171	6.0171	4.2786	5.0971	6.0971	4.2407	5.0571	6.0571
max	4.5429	5.4057	6.4057	4.4257	5.2543	6.2543	4.4729	5.3300	6.3300
min	3.3771	4.0457	5.0457	3.5157	4.2200	5.2200	3.4464	4.1329	5.1329

The TOPSIS method demonstrates the best result in all three cases (best, worst, and best-worst) when the weight of criteria is calculated using the RR method. Indeed, from Table we can write the following relations:

$$\mathrm{TOPSIS}_{\mathrm{RR}}^{\mathrm{best}} = 0.2902 > \mathrm{TOPSIS}_{\mathrm{PD}}^{\mathrm{best}} = 0.2779$$

$$\mathrm{TOPSIS}_{\mathrm{RR}}^{\mathrm{worst}} = 0.3127 > \mathrm{TOPSIS}_{\mathrm{PD}}^{\mathrm{worst}} = 0.0008$$

$$\mathrm{TOPSIS}_{\mathrm{RR}}^{\mathrm{bestworst}} = 0.3092 > \mathrm{TOPSIS}_{\mathrm{PD}}^{\mathrm{bestworst}} = 0.1602.$$

Table 32. The best and the worst ideal solution matrix based on ranking

	Best			Worst			(Best+worst)/2		
	S+	S-	V	S+	S-	V	S+	S-	V
A_1	0.0000	1.2985	1.0000	0.0132	0.9877	0.9868	0.0000	1.1431	1.0000
A_2	0.6010	0.6979	0.5373	0.3769	0.6184	0.6213	0.4857	0.6581	0.5754
A_3	0.9268	0.3765	0.2889	0.7114	0.2875	0.2878	0.8153	0.3320	0.2893
A_4	0.5596	0.7391	0.5691	0.3347	0.6608	0.6638	0.4439	0.6999	0.6119
A_5	0.7754	0.5256	0.4040	0.7033	0.2955	0.2959	0.7356	0.4104	0.3581
A_6	0.7264	0.5726	0.4408	0.5262	0.4695	0.4715	0.6227	0.5210	0.4556
A_7	1.2985	0.0000	0.0000	0.9946	0.0000	0.0000	1.1431	0.0000	0.0000
A_8	0.6040	0.6946	0.5349	0.3573	0.6374	0.6408	0.4771	0.6660	0.5826
A_9	0.3608	0.9377	0.7221	0.0140	0.9827	0.9860	0.1832	0.9601	0.8398
A_{10}	0.3731	0.9254	0.7127	0.1539	0.8408	0.8453	0.2600	0.8831	0.7725

The VIKOR method shows different results depending on the value of λ. Thus, the VIKOR method shows the best result in all three cases (best, worst and best-worst) when < 0.5, the weight of criteria is calculated using the PD method, and when $\lambda > 0.5$, the best result is calculated when the weight of the criteria is calculated using the RR method.

Generally, when analysing all the results, we observe that when calculating the weights of criteria using the RR method, both the TOPSIS and VIKOR methods show almost stable and satisfactory results, regardless of the selection of the best, worst, and best-worst methods. For example, let's look through the results of the TOPSIS method:

$$\mathrm{TOPSIS}_{\mathrm{RR}}^{\mathrm{best}} = 0.2902,$$

$$\mathrm{TOPSIS}_{\mathrm{RR}}^{\mathrm{worst}} = 0.3127,$$

$$\mathrm{TOPSIS}_{\mathrm{RR}}^{\mathrm{bestworst}} = 0.3092.$$

We can observe significant differences (leaps) between the results obtained by the PD method:

$$\mathrm{TOPSIS}_{\mathrm{PD}}^{\mathrm{best}} = 0.2779,$$

$$\mathrm{TOPSIS}_{\mathrm{PD}}^{\mathrm{worst}} = 0.0008,$$

$$\mathrm{TOPSIS}_{\mathrm{PD}}^{\mathrm{bestworst}} = 0.1602.$$

The condition is the same for the VIKOR method. Similar results can be observed for all values of λ.

9. Conclusion and future work

Considering this, the paper examines the assessment issue of research institutions based on not only one, but several criteria. Thus, the main criteria for multi-criteria assessment of research institutions are defined by experts. The article defined a system of criteria for research activity assessment of the institutions and organizations. An approach based on the "worst case" and "best case" methods were proposed for calculating the weights of these criteria. Using the "worst case" and "best case" methods, based on a pairwise comparison of the criteria, the preference degree of other criteria over the worst criterion and the preference degree of the best criterion over other criteria were determined by experts. One of the proposed approaches is based on the weights of criteria using the preference degree, and the other is based on the ranking of criteria based on the preference degree. TOPSIS and VIKOR methods were used for the assessment of alternatives. As a result of the comparative analysis, we determined that the results obtained from the ranking of criteria according to the preference degree showed a stable and satisfactory result. In future studies, we will consider the development of extended models of the proposed approaches.

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