Gulbahar G. Yusufova-Aghabalayeva, Peri M. Isayeva
Institute of Control Systems of ANAS, Baku, Azerbaijan
gulya.a@rambler.ru

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PROBLEM OF THE OPTIMAL DIVISION OF A COUNTRY'S TERRITORY BASED ON POPULATION SIZE

With a view to the formation of territorial administration, mathematical criteria of their geographical boundaries are defined; mathematical solutions to the problems of optimal separation of areas based on the density of population and the definition of administrative centers are given.

Keywords: *territory, population, management, optimal, division, mathematical, model.*

Introduction

The geographical specificity and the sporadically inhabited territory of any country impede the administrative management of a country. It imposes problems for the optimal management of economy, education, healthcare, elections and other areas. Moreover, as the whole country is divided into numerous optimal administrative geographical regions, districts, cities, towns and villages, their management from one center makes the problem more acute.

Effective management of the entire infrastructure of a country depends on the geometric dimensions of these territories, the length of the borders, the number and density of the local population settled in each administrative management territories, and the correct selection of the location of the management centers. Thus, urban, regional, urban, rural, municipal and other governing bodies of each country are formed based on these indicators [1-8].

Nowadays, management issues become difficult, even impossible, without the use of information communication technologies. Taking into account the different features of governance in the management of regions, the automated system management should be, multi-dimensional, multi-level and with distributed structure. Furthermore, the real application of the automated system management in the governance structures of a country will solve the above-mentioned issues much more easily. Evidently, the level of the future system’s structure and information support will depend on the geographical landscape of settlements, the correct distribution of management locations, and at the same time on the human resources of the country. In this regard, the selected case is deemed relevant.

An extensive review of the literature was carried out for the precise solution of the issue [1-10]. The obtained data was analyzed and existing shortcomings were revealed, as a result, the following issue was set.

The statement and status of the problem

The problem comprises the development of a mathematical algorithm for determination of the optimal boundaries of administrative management areas in accordance with the density and the number of residents of a country. As noted, two criteria are used in the process of problem solution: equality in the number of population and regional (neighborhood) administrative borders.

There are related structures for the implementation of the optimal division of the territories, and the reorganization of the distributions within administrative management is regulated by normative documents. The rules of divisions are provided in the legislation of the country and specified in the constitution [1, 2]. The areas of administrative management is often required to be as close as possible to the number of the population. Moreover, administrative and natural boundaries, other geographical characteristics, sparsity of population, isolation and other important factors are also taken into account [3].

One of the important factors that many countries take into consideration is the common interest of the population groups that are geographically located in the same territories. Domestic transport and communication factors are also considered. These factors are of particular importance in the implementation of elections and other processes along with administrative management.

As it is mentioned, the territory of the country, where the population resides, should have territorial boundaries according to the number or density in order to implement effective governance issues. Thus, the population of these territories should be registered based on its resources (parameters), and the optimal boundaries of the settlements should be determined.

First of all, the territories controlled by the governing bodies, the location and geography of its inhabitants should be revealed for the implementation of the control and management function. Consequently, the boundaries of the designated residential areas should reflect the identification of the territories of each local executive and municipal governance.

Obviously, the number of population and the boundaries of settlements change over time, and they are characterized by several significant quantities. Thus, the expansion of the settlements due to the increase of the population's size, and the emergence of new settlements, construction of new residential areas, emergence of new addresses and their replacements, migration, permanent or temporary relocation in case of emergencies or wars, and so forth result in changing the boundaries of settlements. This factor is of particular importance in the registration and is a key indicator in the mobilization process, primarily in management [1-8].

The geographical location of settlements (villages, districts, cities, etc.) differs according to several important features. These features, in turn, emerge the optimal division of the territories covered by separate registration bodies. This is explained by distinctive factors, the most important of which is a human factor. In addition to the individual registration parameters of the people, there are other factors that are considered important for the local governance authorities of the same settlements. These are called non-registration parameters [3, 7, 8].

The borders of regular settlements (RS), geographical location, natural resources, conditions and so forth can be considered as key indicators of living standards of the population. Effective and efficient use of available natural resources in this area is the main task of the local executive power and even the governmental authorities of the county. Obtaining operative and uninterrupted information about the relevant settlements and their residents that are subordinated to them leads to the effective resolution of the problems of citizens. At this point, optimal division of the territories and human factors are of particular importance.

This article examines the mathematical-statistical division of the borders of RSs of the country, and provides its mathematical solution. The efficiency of the applied mathematical model determines the effective management of the population in the registration process as a result of their flexible operations over the territory and high percentage of the registry. This, in turn, depends on the optimal and precise determination of the boundaries of the settlement.

When determining the boundaries of the registration stations (BRS), the geometrical dimensions and shape of these areas are of great importance. In addition, the length of the boundaries complicates this issue. Thus, the mathematical model of the optimal definition of the boundaries of the registration areas or stations should be determined as a result of the statistical assessment, the compactness of the territories, the specific features of the vast set of the regions where the records are located, and other important factors.

At present, this process is being implemented in a new dimension; the State Cadaster Service carries out precise division and registration of the territories and lands with the application of advanced technologies. In each case, the principle of optimal measurement and division should be scientifically verified and based on mathematical models and calculations.

The definition of BRS should be viewed as mathematical collections or as a set. In this case, the boundaries of the territory correspond to the boundary of each mathematical collection, whereas the elements of the mathematical collection refer to the parameters of those living within those boundaries. Thus, each administrative district or region is a separate sub-division, and in general, all regions represent the whole country, its entire territory and citizens [1-3].

Depending on the nature of the registration system, the optimal determination of the boundaries and dimensions of the territories is of particular importance. The inhabitants of each area should be taken as a group, and the relation of their elements with the elements of this or that group should be taken into consideration. The registration systems of the main cluster elements must be linked to one another in the information space. In this case, the status of the elements of the sub-groups can change generating a mixed process which will complicate the problem. Thus, every citizen of the country must be determined by a certain address, i.e. he/she must belong only to one sub-group. Here, the evaluation status of each individual is determined according to his/her registration and non-registration parameters during the division and management.

Certain executive structures and regulations should be available for the realization of determination or re-division of the borders of RS. Different collections of settlements generate different registration results, and key personal (individual) examples play a crucial role. Formal structures are shaped in the course of registration, which are based on the following rules:

1) Will citizens expand the lines (addresses) or boundaries of RS?

2) Citizens will not change their registration parameters or be extremely responsible for their registration.

3) Those who define the lines or borders of RS should be politically neutral and not dependent on the legislative authority.

4) Legislative authorities should not interfere with the process of the division of the territories.

5) Public access should be provided to some division mechanisms or processes.

6) The lines (borders) should be optimal to monitor the criteria for the division of the territories. In this case, the criteria must be acceptable.

7) The division of the district territories should be regularly restored, the duration of the division process should be determined, and so forth.

Evidently, the division of the territories is a very complicated issue.

Determination or division of the boundaries across the registered territory of the country is carried out in accordance with the "Structures and Regulations for the Estimation of Border Registrations". This process is guided by the formal structure and rules (management). During the execution of the process, every step of the area is recorded; division schedule (the issues involved in drawing up the boundaries of the registered area) is described and estimated; and finally, division and database creation is completed. In addition, the identification of disapproved boundaries and areas are discussed for the registration management.

The experience of the developed countries in the definition and division of boundaries significantly differs and changes over the time. It should be noted that since these distinctive features do not have common concepts, it is necessary to generalize them for the given division method and principle to have multi-system function. The universal way of determination and division of the boundaries is to address it on a scientific basis. The governing authorities of the country disagree with a number of substantive division problems, and are persisting in carrying out an impartial and independent division process. However, there are three general principles adopted:

– being a leading region (advantages);
– equality in the approval of borders;
– mutual relations.

The boundaries of the registered area should be drawn up enabling the elements (citizens) to be provided with high-level management and care. This, in general, means the maximal satisfaction of the interests of societies (communities) formed within the borders. The interest (percent) of the societies (communities) can be determined by the sets of methods. For example, they may include natural societies (communities), ethnic and administrative territories, neighbors or territories surrounded by actual boundaries. If the regions (territories) do not represent the societies (communities) of citizens and their interests, and even, if each citizen does not represent his/her registration area, this registration system has not been properly established and its operation is not valid.

Regardless of the characteristics or political convictions of the citizens, if the services provided to the citizen in the registration area are satisfactory and the employees of that legislative body do their best to protect their interests (percent) in the registration territory, the registration system and its defined territory effectively operates.

In order to ensure the relative equality and acceptable relations of the population in the registration area, the precise boundaries must first be determined and drawn. Districts (territories), which are densely populated, will be loaded equally in the registration process and their management will not be complicated. For example, if a person is settled in another region (territory), which is twice more populated than his/her previous region, the satisfaction rate of his/her interest will be reduced twice. Hence, the smaller the registration area and the number of citizens, the more effective the management will be. On the other hand, dividing the areas into too small territories often complicates the structure of the registration system.

In this case, the territory of the country or the registration area is called non-proportional and very changeable and all citizens have the equal weight functions.

Thus, in terms of optimal division of the territories, the main criterion is the determination of the registration and non-registration parameters of the population and territories and the assessment of their integral indicators.

In order to have the same regulations of regional and governance processes, the procedure of bordering should be regulated by law and should not depend on the executors. If the division process is regulated by law, then all stakeholders should refrain from trying to effect the outcome. Otherwise, if the stakeholders (enterprises) interfere the process due to their concern, then all parties should be provided with the access to the process. If the legislature is an interested party that receives the set (plurality), then it will be able to manage the bordering process. These guidelines must be clear and agreed by all key stakeholders, and their mutual involvement in the division process should be ensured.

When identifying the equal size of the registration areas, the territories should represent an equal number of citizens to meet the requirements of equality of the citizens in the information and registration system. Of course, only in ideal case, the mathematical equality can be achieved, and the registration systems should be democratic and ideal. The inclination from the average value of the number of citizens of the optimally divided territory is regulated by the legislation ($\leq$ 5%, and in exceptional cases $\leq$ 10%).

The division of all settlements into registration areas is called territorial division (TD). As a rule, it is often solved before pre-registration campaign.

Numerous studies have been conducted so far for the identification of various aspects of the problem of land division and for the definition of its boundaries [1, 2, 4, 5].

Overall analysis of the world practice shows that the definition of the borders of the settlements is an important part of the registration system. As a rule, the borders are not set once. It is necessary to adjust the boundaries of the territories over time, which is at least due to the demographic changes.

The study of the establishment of settlements, two factors are of particular importance:

1) dimensions (size) of the settlements;
2) balancing the settlements with existing administrative-political boundaries.

Criteria for the division of the territories are often specified in the registration legislation of the country. They usually cover the following factors:

• Equal number of the population;

• Taking into account other geographical features, such as regional and local administrative boundaries and natural boundaries;

• Recognition of the existing of the population groups with common interests.

A proper government body should be established to implement the settlement division. Typically, the regular division is implemented by a certain rule. For example, in the US. this interval is 10 years, in Australia - 7 years, and in New Zealand - 5 years. As a rule, the term of the division process is also determined: no more than 2 years in the US and 4 years in the UK.

The public involvement in the division process of the settlements is also an important issue, and public opinion is taken into account.

Mathematical solution of the optimal division of territories

Assuming that the set *I* consisting of *n* number of settlements with $p_i,\ i=1,...,n$ number of population, which interact by a certain rule

$$P=\sum_{i=1}^{n} p_i \quad (1)$$

Considering all settlements as a set *I*, it is divided into *N* number of subsets $I^1, I^2, ...I^N$. Obviously, $N \le n$. In this case, the following is obtained:

$$I=\bigcup_{k=1}^{N} I^k,\ I^i \bigcap I^j = ø, \qquad i \ne j \quad (2)$$

The appropriate subsets should be divided with $P_1, P_2, ..., P_N$ number of population in each in order to achieve the slightest difference from the average value accepted for the territories in each sub-sets, that is

$$\widetilde{P} = P/N$$

This division can be mathematically explained as follows:

$$F_1(I^1,...,\ I^N)=\max_{i=1,N}\left|P_i-\widetilde{P}\right| \to \min, \tag{3}$$

or

$$F_2(I^1,...,\ I^N)=\sum_{i=1}^{N}\left|P_i-\widetilde{P}\right| \to \min, \tag{4}$$

or

$$F_3(I^1,...,\ I^N)=\sum_{i=1}^{N}(P_i-\widetilde{P})^2 \to \min \tag{5}$$

Choosing any of the criteria (3) - (5) defines the optimization from different classes and depends on the preference of the determination of the optimum of the problem. Thus, each of the criteria (3) - (5) defines different optimal solutions within other equal conditions.

Division into sub-sets is realized taking into account the conditions that arise from the administrative and cultural division of the republic, including the geographical location of the settlement, transport links and so forth. Each of these factors has a different effect on the inclusion of the settlement into this or that area. For example, the impact rate is informally determined by the values of relationships. In specific cases, an expert appraisal apparatus is involved in establishing the relationship parameters of the settlement sets in the country.

Statement of the mathematical problem of the division of territories (DT).

The linked graph with the orientation and weight in the form of the set with common vertices $E=\{E_i,\ i\in I\}$ and arcs $V=\{v_{ij};\ i,j\in I\}$ is presented.

Assume that the weight of the i-th vertices is p_i, the conflict matrix

$$V_{ij}=\begin{cases}0, \text{if the } \iota\text{-th and } j\text{-th vertices are not connected;}\\ v_{ij}>0, \text{if the } i\text{-th and } j\text{-th vertices are connected.;}\end{cases}$$

determines the connectivity of the graph.

Here V_{ij} is the weight of the arc connecting the *i*-th and *j*-th vertices. It should be noted that generally:

$$V_{ij} = V_{ji}; \ \ i, j \in I$$

In other words, the weight of the arc depends on its direction. The quantity v_{ij} characterizes the degree of desirability of the use of the *i*-th and *j*-th vertices in the same subgraph when dividing the entire graph into the subgraphs.

Obviously, DT problem is equivalent to the division of given *N* number of connected subgraphs (E^k, V^k), $k = 1,..., N$, taking into account the possible signs and limitations on the division options. One of the quality indicators of the division characterized by the relations (2) - (5) can be accepted as DT indicator.

Consideration of the connectivity of the graph can be carried out at the level of criteria or restrictions. Assume that the sub-graphs (E^k, V^k), $k = 1,..., N$ are defined as follows:

$$(E,V) = \bigcup_{k=1}^{N}(E^k, V^k),$$

$$E^k = \{e_i^k;\ i \in I^k\},\ \ I = \bigcup_{k=1}^{N} I^k,$$

$$V^k = \{v_{ij}^k; (i, j) \in I^k\} \quad k = 1,..., N.$$

The total weight of the arcs of the k -th sub-graph is

$$Q^k\left(I^k\right) = \sum_{(i,j)\in I^k} v_{ij}^k \tag{6}$$

In other words, $Q^k\left(I^k\right)$ shows the dependence of the weight of the arcs on the vertices included to the k -th sub-graph.

Then, there may be restrictions for the DT problem:

$$Q^k\left(I^k\right) \le Q^{given}, \qquad k = 1,..., N, \tag{7}$$

Here, Q^{given} is any predetermined quantity defined by the expert and which characterizes the possible value of the weight of undesirable relationships in the subgraphs.

The DT problem should be mathematically divided by *N* number of subgroups $I^1,..., I^N$ of the set *I*, then

$$\text{if } I^i \cap I^j = \varnothing,\ i \ne j,\ \ i, j = 1,..., N \tag{8}$$

the condition

$$\bigcup_{k=1}^{N} I^k = I \tag{9}$$

is provided, and when the restrictions (7) are provided, the value of any of the criteria (3) - (5) equals to minimum (problem 1).

The DT problem can be set as a multi-criterion optimization problem with the restriction criterion (7). Then, the vector of criteria is defined as follows:

$$\left(F, Q^1(I^1),..., Q^N(I^N)\right) \to \min, \tag{10}$$

or

$$\left(F, \max_i [Q^i(I^i)]\right) \to \min. \tag{11}$$

Here, F is one of the above mentioned criteria (3) - (5) (problem 2).

Including the following variables to the first and second problems, they can be denoted as the one-criterion and multi-criterion discrete optimization (DO) problems:

$$x_{ij} = \begin{cases} 0, \\ 1, \end{cases} \text{ if the } i\text{-th vertice does not refer to the } j\text{-th sub-graph;}$$

if the i-th vertice refers to the j-th sub-graph.

In this case, the DT problem becomes a special case of classic "portfolio" and division problems. If the size of the DO problem equals to $N \times n$, and if N is close to 100, and n - to 1000, its solution through common DO methods is not possible even with advanced computing tools.

Given the fact that the expert information on the graphs and their weight is often fuzzy, it is even more correct to designate the DT problem as a decision-making problem in the fuzzy set.

Assume that the functions $\mu_{I^k}(x)$ are given, which determine the degree of belonging to the elements $x \in I$ to the sub-set I^k $(r = 1,..., N)$. The degree of belonging of all vertices to the k - the subset I^k is determined by the following functions:

$$P_1^k = \min_{x \in I^k} \mu_{I^k}(x)$$

or

$$P_2^k = \sum_{x \in I^k} \mu_{I^k}(x)$$

The degree of belonging of the elements of the subset I^k to this sub-set is determined by evaluation of the connectivity of only the vertices with one another.

In the fuzzy statement, the DT problem includes the optimization of one of the criteria (3) - (5) in the context of mixed conditions:

$$\min_{1 \le k \le N} p_1^k \ge \alpha_1$$

or

$$\sum_{k=1}^{N} p_2^k \ge \alpha_2$$

where, α_1, α_2 are the predetermined values of the probability rate for the problem solution. Obviously, the bigger the value, the “worse” the optimal values of the respective criteria will be.

Note that the functions $\mu_{I^k}(x)$ are based on the expert information about the possible values of the weights of the arcs.

One fact should be taken into consideration in any statement of the DT problem. According to the legislation, the inclination from the average value $\tilde{p}$ in each area should not exceed 5%. However, in exceptional cases, this figure may vary from $\tilde{p}$ by 10% in some areas.

Thus, the following restriction should be taken into account in all statements of the DT problem:

$$|p_i - \tilde{p}| \le \Delta p, \qquad i = 1, \dots, N$$

The review of the principle of division of the territories shows that Δp is the permissible inclination of the average number of population on the settlements permitted by the legislation.

Thus, the goal of the DT problem is to divide a set of settlements n into N number of sub-sets containing maximally equal number of people taking into account already formed traditional, administrative, cultural and geographical factors.

Over time, a need for a change in the number of divided territories arises and, accordingly, the changes are made to the previously designated DT.

The following initial information should be provided for the solution of the DT problem:

1) number of the settlements (n);

2) code, name, number of citizens of the settlements $(p_1,..., p_n)$;

3) connectivity coefficient, which takes into account the desirability, and antipathy of the probability and impossibility of inclusion of the settlement to the same area as the neighboring stations $(v_{ij},\ \ i, j \in I)$;

4) number of organized areas (*N*), if possible, can be shown as the center of the settlement;

5) borders of each settlement should be shown on the electronic map for its illustration in the graph results.

Thus, the database containing information about the names of the territories, the number of population and the names of the settlements included in each area, and so forth is created. If necessary, digital information is called, which can graphically describe the optimal map of the area on the computer.

Let's look at the optimal division algorithm of the service centers (SC). Information and registration systems of the population are composed of subsystems, covering a specific settlement or administrative management territory. In this case, optimal division of SC is of great importance. In addition to the operative and reliable data collection, processing and storage, the use of this information by the users (citizens) and establishment of their communication with the system is very significant. Thus, the mathematical solution of SC location is also considered in this study case.

The statement of the problem

Assume that there is a set *I* consisting of interconnected *N* stations. Each interconnection is evaluated by $a_{ij} > 0$ showing the value of the relationship between the stations *i* and *j* of the set *I*. If $a_{ii} = 0,\ \ i \in I$, $a_{ii} = 0$ is accepted. It is necessary to allocate *n* number of stations called "centers" from the set *I*, in order to cover all the stations of the set *I* with its surroundings with the same dimensions $A_i,\ \ i = 1,...,n,\ \ n \le N$. The term "surroundings" means the maximum value of the relationship between the center and all the settlements around, or the value of the relationship between the center and all the settlements around. The average value of the relationship of the surroundings can also be expressed. Each of such expressed surroundings provides different definition of the optimum of the centers, and accordingly, chooses different stations of the set *I* as the optimal centers. It is important to note a fact related to the number of centers *n*. Obviously, *n* is smaller than *N*. However, their number can also be determined by the optimization conditions. Optimal surroundings of the centers are implemented by limiting the maximum value of the connections or by limiting the total or average value of the connections within each surrounding. Thus, two-level optimization problem is obtained: one-dimensional optimization problem in relation with *n* is solved at upper level, and at the low level, the centers covering the set *I* for each *n* number of centers as a whole and the surroundings of each of them are defined.

Mathematical statement of the problem

The relationships include a set *I* consisting of the stations *N*, which have the values $a_{ij} > 0\ (i = \overline{1, N};\ j = \overline{1, N})$. It is necessary to choose *n* number of stations $P_1,..., P_n$ out of them as the center and the corresponding subsets of the stations $I_1,..., I_n$ in order to achieve the followings:

$$P_i \in I_i, i = 1,..., n$$

$$P_{ij} \in I_i,\ j = 1,..., m_i, i = 1,..., n$$

$$\bigcup_{i=1}^{n} I_i = I$$

$$I_i \cap I_j = \varnothing, \quad i \neq j$$

where $P_{ij}, (i = \overline{1,n};\ i = \overline{1,m_i})$ are the stations belonging to the i-th center $P_i, (i = \overline{1,n})$, $j = 1,...,m_i$, and m_i - the number of stations belonging to the i-th center, $i = 1,...,n$.

Here, division into subsets can be implemented, taking into account any of the following optimization criteria:

$$\min_{I_1,...I_n} \max_i \sum_{j \in I_i} a_{ij} \qquad \text{(I criterion)}, \tag{12}$$

$$\min_{I_1,...,I_n} \max_i \max_j a_{ij} \qquad \text{(II criterion)}, \tag{13}$$

$$\min_{I_1,...,I_n} \max_i \frac{\sum_{j \in I_i} a_{ij}}{m_i} \qquad \text{(III criterion)} \tag{14}$$

Note that, in this case, an option should be accepted to provide the following restriction:

$$\max_i \sum_{j \in I_i} a_{ij} \leq A \quad \text{(IV criterion)}, \tag{15}$$

where A is a predetermined quantity (for example, the maximum probable time required to reach the most remote point from the central station of the chosen set), and then the respective problem becomes the problem of searching for the point in the probable area. The quantities a_{ij} can be fuzzy and set by experts. Then the appropriate problems will be set as fuzzy optimization issues with the restrictions (12) or (13).

The problem attached to (12) and (13) defines the minimum number n and respective $I_1,...,I_n$-s to provide the following conditions:

$$\max_i \left\{ d_i : d_i = \max_{j \in I_i} a_{ij} \right\} \leq D, \tag{16}$$

where D is a predetermined quantity. This problem includes any of the problems (4) - (7) described above as a sub-problem.

Let's define SC's assignment problem within the theory of discrete optimization.

Let's include the variables matrix:

$$X = ((x_{ij})),\ i = 1,...,n,\ j = 1,...,N$$

$$x_{ij} = \begin{cases} 1, & \text{if the } i\text{-th station follows the } j\text{-th station;} \\ 0, & \end{cases}$$

otherwise

Then, SC's assignment problem can be expressed as follows, by using the criteria (12):

$$\min_{x_{ij}} \left\lfloor \max_{1 \leq i \leq n} \sum_{j=1}^{N} a_{ij} x_{ij} \right\rfloor \tag{17}$$

$$\sum_{j=1}^{N} x_{ij} = 1, \quad i = 1,...,n, \tag{18}$$

$$x_{ij} = 0 \vee 1 \tag{19}$$

The condition (18) indicates that each station is served by only one center. It proves that all the stations are served. The condition (19) shows that the optimized parameters are "Boolean" variables.

The definition of SC by the criterion (13) - (15) and two-level problem (16) are also expressed similarly.

For comparison, the theory of graphs was also considered for the solution of these problems [1, 2, 5, 6]. Assuming that the non-directional weight graph (*E*, *V*) is given, where *E* is the number of vertices in the graph, *V* – the arcs with the graph weights $((a_{ij}))$. Here, a_{ij} is the weight of the acres connecting the *i*-th and *j*-th vertices. It is required to divide the graph into *n* number of interconnected subgraphs, and a vertice (center) is assigned to each of them, and (12), (13), (17) or (15) become valid.

Each of the abovementioned problems belongs to one or another class, each of which has specific research and solution methods.

The measurement problems in their solution include: if *n* is a few thousand and *N* is a decimal number, the problem solution is practically impossible without considering the specific features mentioned in all versions of the problem statement. Because, these problems are related to the combination type counting, their solutions refer to the most complex problems, namely the *NP*-class. The initial information to address the SC problem includes the followings:

1) Total number of the settlements - *N*;
2) Number of SC to be created (if provided) -*n*;
3) Connection values between all stations a_{ij}, $i = \overline{1, n;}$, $j = \overline{1, N}$ (if the stations, where SC are located, are provided, it is necessary to set the stations to serve, however, their connection values are enough to be mentioned). Location of each station in the electronic map for the graphic illustration of the results.

As mentioned above, in the division of the territories and SC's placement, the statement of the optimization problem may vary depending on the accepted criteria and considered limitations. Regardless of their statement, these problems are combined for the integer arguments, the finite destination area, and the large size.

According to the theory of integer programming problems, it is known that such problems refer to the integer *NP* class, in other words, their solutions require selection of all options.

Given the real dimensions of the above-mentioned problems, even if we take into account the power of modern computer technology, selection of all options separately is impossible. To solve this type of problem, typically, heuristic algorithms should be developed, taking into account the specific features of the problems and the solution experience of small sized problems.

As a result of the solution, the names (codes) of *n* number of stations defined as the information-registration centers in the form of output information a file with a special structure, including the names (codes) of each station attached to each center are grouped in the form of a specifically structured file. If necessary, this information can be graphically visualized on the electronic map by recalling the numerical information.

It should be noted that such optimal solution of the territorial division of the country into a separate administrative management area consolidates the principles of effective state territorial governance and creates a basis for the economic and political development. It also ensures a favorable condition for the employment of the population, planning, operational and reliable management in all areas. Moreover, the proper division of the territories defines the structure, volume and functions of the information and registration system, including the selection of its information and communication tools and real-time operation. Accordingly, the optimal structure of the developed system and reliable information support are implemented.

Conclusion

The mathematical criteria of the administrative borders in the residential areas were defined, and the mathematical model of the optimal division of the territory according to the number of citizens was developed. In addition, the mathematical statement, mathematical expression and goals of the problem of optimal division of the administrative governance territories were defined. Moreover, the mathematical model for the equal distribution of the citizens on registration areas was developed.

Furthermore, the primary information for the mathematical statement and solution of the problem of optimal positioning of the SC was identified, and the algorithm for the identification of their location was developed.

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