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APPLICATION OF NEURAL NETWORK TO SHAPE OPTIMIZATION PROBLEM

In the present paper we consider shape optimization problem and reduce it to the integer programming by discretization of the original problem. This formulation of the considered problem allows applying the neural networks for its solving.

***Keywords**: shape optimization, artificial neural network, approximate theory, network training.*

1. Introduction

Identification of non-linear systems is an important task for model based control, system design, simulation, prediction and fault diagnosis. For investigating this kind of question or problem, we can use neural networks. In recent years artificial neural networks have gained a wide attention in control applications. It is the ability of the artificial neural networks to model non-linear systems that can be the most readily exploited in the synthesis of non-linear controllers. Artificial neural networks have been used to formulate a variety of control strategies [1, 2].

In present paper, we consider minimization of the domain dependant integral functional. But since such type problems are shape optimization problems, their investigation is connected with some mathematical difficulties [3–7]. This difficulty is related mainly to the mathematical definition of the variation of the domain characterized by its boundary variation. The new approach introduced in [5–7] tends to avoid these difficulties. It consists of representing a convex domain by its support function. The variation of the domain then is naturally replaced by the variation of the corresponding support function. In the numerical simulation process after each iteration we get not only a set of boundary points, but also a support function. The domain is reconstructed as a sub-differential of its support function at the point 0 [8]. We see this technique allowed to solve such type problems when the considered domains are convex. But, convexity of the domains is a very hard condition from the practical point of view. Therefore, in order to solve this problem we reduce it to the integer programming problem by its discretization. We applied a neural network for solving the integer programming problem.

2. Statement of the problem

By K we denote a set of domains $D \in R^2$. K may be determined for example, by fixing the area, the length of the boundary or by the relation $D_0 \subset D \subset D_1$ and etc. Here $D_0, D_1 \subset R^2$ are the given domains. Suppose that there exists a rectangle Q such that if $D \in K$, then $D \subset Q$. We consider the following functional

$$J(D) = \int_D f(x)dx \tag{1}$$

Here, $f(x)$ is a function continuous on Q. If $f(x) \equiv 1,\ x \in Q$, then $J(D) = mesD$.

Let it be required to minimize the functional (1) on the set K. For solving of this problem we discretize the rectangle Q with respect to uniform small step $h>0$. We denote the obtained lattice by Q_h.

3. Discretization and solving of the problem

It is clear that in this case we can oppose to each $D \subset Q$ its discrete analogy $\Omega \subset Q_h$ lattice. By d_{ij}, $i=\overline{1,m_1}$, $j=\overline{1,m_2}$ we denote the square of the lattice Q_h corresponding to the ith row and jth column. Here m_1 and m_2 are the numbers of vertical and horizontal partition points, respectively. Now, we discretize the functional (1).

We denote

$$H_D(x)=\begin{cases}1, & x\in D\\ 0, & x\notin D\end{cases}.$$

Then, it is clear that

$$J(D)=\int_Q f(x)H_D(x)dx\,. \tag{2}$$

We introduce the following denotation

$$z_{ij}(\Omega)=\begin{cases}1, & d_{ij}\in\Omega\\ 0, & d_{ij}\notin\Omega\end{cases}.$$

Then we can discretize functional (2) as follows

$$I(\Omega)=h^2\sum_{i=1}^{m_1}\sum_{j=1}^{m_2} f_{ij}z_{ij}(\Omega) \tag{3}$$

Here, f_{ij} are the values of the function $f(x)$ at any point of the square $d_{ij}\in Q_h$. In this case, we can give the discrete analogy of the restriction $\Omega\in K_h$. For example, if the section K is a set of domains whose area equal the number c, then its discrete analogy consists of the set of lattices Ω satisfying the condition

$$\sum_{i=1}^{m_1}\sum_{j=1}^{m_2} z_{ij}(\Omega)=\frac{c}{h^2} \tag{4}$$

Here we'll assume that h is such a step that the number $\overline{c}=\frac{c}{h^2}$ is natural. In the same way, the other restrictions also may be written in the discrete form. For definiteness, suppose that the restricition K is as $mesD=c$.

Thus, the problem on minimization of functional (1) provided $D\in K$ is reduced to integer programming problem

$$\sum_{i=1}^{m_1}\sum_{j=1}^{m_2} f_{ij}z_{ij}(\Omega)\to\min \tag{5}$$

$$\sum_{i=1}^{m_1}\sum_{j=1}^{m_2} z_{ij}=\overline{c} \tag{6}$$

$$z_{ij}\in\{0,1\} \tag{7}$$

To solve the integer programming problem (5) – (7) we can use the MATLAB program packets. Assume that we have solved problem (5) – (7) and found the variables z_{ij}, $i=\overline{1,m_1}$, $j=\overline{1,m_2}$. Then the approximately solved lattice Ω is defined as follows:

$$\Omega = \{d_{ij} : z_{ij} = 1, i = \overline{1, m_1}, j = \overline{1, m_2}\} \tag{8}$$

Reduction of minimization of domain dependent functional (1) to problem (5)-(7) allows to apply the artificial neural network to solve this problem. It is seen, having given the $m_1 \times m_2$ dimensional matrix $F = (f_{ij})$, $i = \overline{1, m_1}$, $j = \overline{1, m_2}$, we get the matrix $Z = (z_{ij})$ as a solution of problem (5) – (7). In the solution of problem (5) – (7) matrix Z changes due to change of the matrix F. So the lattice Ω giving minimum to functional (1) also changes.

Using this fact we give the scheme of application of the artificial neural network to solve the problem on minimization of functional (1).

1. The problem on minimization of functional provided $D \in K$ (1) is reduced to integer programming problem (5) – (7).
2. Having solved problem (5)–(7) for N number matrices $F_1 = (f_{ij}^1), F_2 = (f_{ij}^2), \ldots,$ $F_N = (f_{ij}^N)$ we find the matrices $Z_1 = (z_{ij}^1), Z_2 = (z_{ij}^2), \ldots, Z_N = (z_{ij}^N)$.
3. We must create such an artificial neural network that it could associate the output matrices $Z_1, Z_2, \ldots, Z_N$ to input matrices $F_1, F_2, \ldots, F_N$. Assume that the artificial neural network corresponding to these input and output has been established. Denote this artificial neural network by $C(N)$.
4. Assume that our aim it is find the matrix $Z = (z_{ij})$ being the solution of problem (5)-(7) according to the given matrix $F = (f_{ij})$, $i = \overline{1, m_1}$, $j = \overline{1, m_2}$. For that we give the matrix $F = (f_{ij})$ as input variables of the constructed artificial neural network $C(N)$. As an output variable this lattice $C(N)$ will give us a new matrix $Z = (z_{ij})$. We'll accept this matrix as an approximate solution of problem (5)-(7).
5. The lattice Ω corresponding to the matrix $Z = (z_{ij})$ is established by formula (8).

Notice that the establishment of the lattice corresponding to the matrix may be realized by the application of the artificial neural network. Notice that as the number N of the input matrices increases, the exactness of an approximate solution determined by the artificial neural network will be also improved.

Now, using this scheme, we solve problem (1) on some model example and analyze the obtained results.

Example. Let $f(x) = \dfrac{x_1^2}{16} + \dfrac{x_2^2}{4} - 1$. In this case solution of the problem (1) is ellipse with axes $a = 4,\ b = 2$. Applying the considered algorithm, we obtain N=15 (figure1) and N=25 (figure 2).

Thus, applying a neural network we can solve the shape optimization problem approximately and as seen this solution is close to the exact solution.

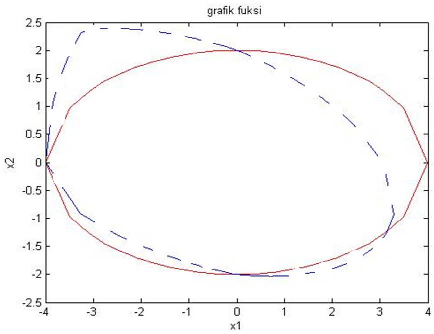

Figure 1. N=15

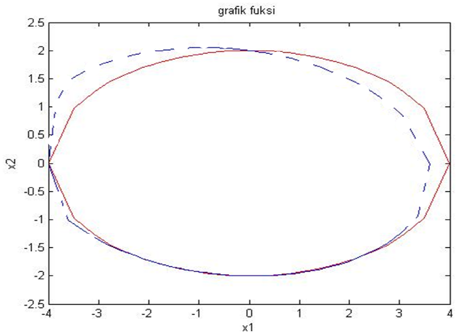

Figure 2. N=25

4. Conclusion

Using new approach shape optimization problem reduced to integer programming problem. This allowed applying neural networks to solve the problem. We considered some model examples which show the effectiveness of the proposed approach.

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Formaya görə optimallaşma məsələsinə neyron şəbəkənin tətbiqi

Məqalədə optimal formanın tapılması məsələsinə baxılır və məsələ diskretləşdirilərək tam qiymətli optimallaşma məsələsinə gətirilir. Məsələnin belə qoyuluşu onun həllinə neyron şəbəkənin tətbiqinə imkan verir.

***Açar sözlər**: forma optimallaşma, süni neyron şəbəkələr, aproksimasiya nəzəriyyəsi, şəbəkənin öyrədilməsi.*

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Применение нейронной сети к задаче оптимизации по форме

В настоящей работе рассматривается задача нахождения оптимальной формы, и она с дискретизацией сводится к задаче целочисленного программирования. Такой подход позволяет применить нейронные сети для решения поставленной задачи.

***Ключевые слова**: оптимизация формы, искусственные нейронные сети, теория аппроксимации, обучение сетей.*